Open problems for YR-CONCUR 2022

Problem 3 (from Arka Ghosh)

Title: Minimal automata for a given transformation monoid.

Description:
I got to know about the following problem in different variations from Szymon Toruńczyk and Ismaël Jecker. In automata theoretic terms the problem asks that given an automaton M with transformation monoid T_M, if there exists a smaller automaton N such that T_N is isomorphic with T_M. This problem is clearly decidable, but the exact complexity of this is unknown. This problem can also be written in the following way. Let F_n be the monoid of all functions from $\{1,\dots,n\}$ to itself. Given $H \subseteq F_n$ is it possible that the submonoid generated by H is isomorphic to a submonoid of F_m for some $m \leq n$.

Problem 4 (from David Purser)

Title: Universal coverability for Orthant VAS

Description:
In a vector addition system sum vectors taken non-deterministically from a given set, providing the sum remains in the positive orthant (all vectors positive). Z-VAS can visit all orthants, but the set of vectors that can be applied is the same in every orthant.

An orthant VAS has a different set of vectors for each orthant. Any vector from the set of the current orthant can be applied, the result of which may be in the same orthant, or another orthant.

The sets of vectors are monotonically increasing; so that if a vector can be applied in some orthant, it can also be applied in every orthant with additional positive dimensions.

The coverability problem asks whether the positive orthant can be reached from a given point and is undecidable. Thus too is reachabilitiy.

The universal coverability problem asks whether the positive orthant can be reached from every starting point. Is known to be decidable in dimension 3. Question: can this be extended to dimension 4+, and/or is it also undecidable in general?

Problem 5 (from Christoph Haase)

Title: The Complexity of Presburger Arithmetic with Stars

Description:
Let M be a set of tuples of natural numbers in N^m, and define

M* := { v_1 + .. + v_k : k\ge 0, v_i \in M, 1\le i\le k }.

We can syntactically extend Presburger arithmetic with stars by allowing for formulas F*(x_1,..,x_m), where F is a formula (which may contain stars itself). If F(x_1,..,x_m) defines the set M then F*(x_1,..,x_m) defines the set M*.

It is known that the the existential fragment of Presburger arithmetic with stars is NEXP-complete in general, and NP-complete for fixed star-height [1]. However, only a non-elementary upper bound for deciding the full first-order theory is known, and no harder lower bound than the classical lower bounds for Presburger arithmetic is known. The precise complexity of Presburger arithmetic with stars has remained a wide open problem.

[1] Haase, C. and Zetzsche, G., 2019, June. Presburger arithmetic with stars, rational subsets of graph groups, and nested zero tests. In 2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS) (pp. 1-14). IEEE.

This problem is also mentioned as an open problem in the concluding remarks in:

Chistikov, D., Haase, C. and Mansutti, A., 2022, August. Geometric decision procedures and the VC dimension of linear arithmetic theories. In Proceedings of the 37th Annual ACM/IEEE Symposium on Logic in Computer Science (pp. 1-13).

Problem 6 (from Wojciech Czerwiński)

Title: The Reachability Problem in Fixed Vector Addition Systems

Description:
The problem was mentioned to me by Georg Zetzsche, then I worked on it with Ismael Jecker and mentioned it to many other people.

It is known that the reachability problem for VASes (vector addition systems) is Ackermann-complete. In this problem we are given a VAS V together with its source and target configurations and we ask whether there is a run from the source to the target in V. We can ask the same question for a fixed VAS V. Of course for some VASes the problem is simple, for example for a VAS with no transitions there is a run from the source to the target if and only if the source and the target are equal. However we might at first expect that for some VASes the problem will be hard, close to Ackermann-hard.

The conjecture is that it is actually not the case and the problem is in PSpace (or even lower). More concretely the conjecture is that if there is a run from the source to the target then there is also one, which is of length linear wrt. size(source) + size(target). Of course the constant in this linear function may depend on VAS V in a non-elementary (or higher) way. The open problem is to prove or disprove the conjecture.

The special case of VASes with semilinear reachability relation was solved by Jerome Leroux in the paper "Acceleration For Presburger Petri Nets" on VPT@CAV 2013.