Logical Characterization of Hereditary History-preserving Bisimulation over Higher Dimensional Automata

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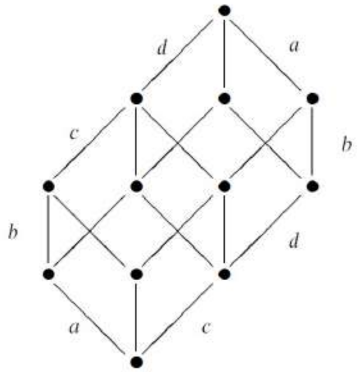
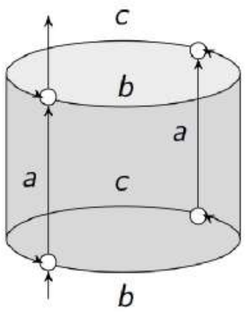
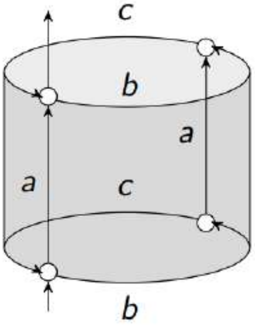
Higher Dimensional Automata

Intuitive definition of Higher dimensional automata



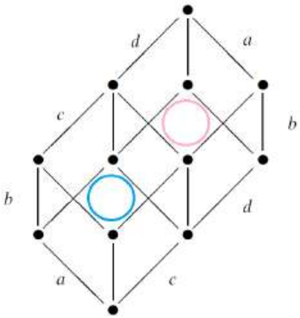


Higher dimensional automata in terms of concurrency

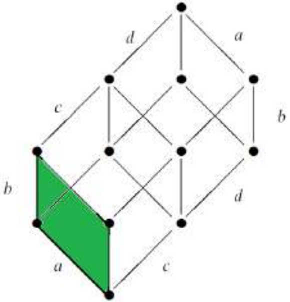


$$a \,||\, (bc)^*$$

Higher dimensional automata in concurrency



Higher dimensional automata in concurrency



a || b

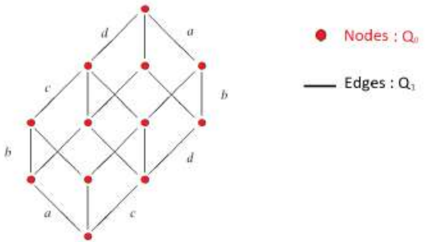
Definition of Higher Dimensional Automata

Definition

A cubical set consists of a family of sets $(\mathbf{Q}_n)_{n \geqslant 0}$ and for every $n \in \mathbf{N}$ a family of maps $s_i, t_i \ : \ \mathbf{Q}_n \to \ \mathbf{Q}_{n-1}$ for every $1 \leqslant i \leqslant n$, such that

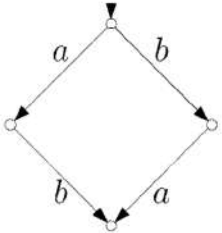
$$\alpha_i \circ \beta_j = \beta_{j-1} \circ \alpha_i \text{ for all } 1 \leqslant i < j \leqslant n \text{ and } \alpha, \beta \in \{s, t\} \quad (1)$$

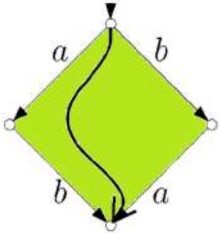
An HDA, labelled over an A, is a tuple $(\mathbf{Q}, s, t, I, F, \lambda)$ with $\mathbf{Q} = \bigcup_{k=0}^{\infty} \mathbf{Q}_k$, $(\mathbf{Q}, s, t)$ a cubical set, $I \in \mathbf{Q}_0$, $F \subseteq \mathbf{Q}_0$.
s and t denote the families of all maps s_i and t_i



Expressiveness power of Higher dimensional automata

Higher Dimensional Automata distinguish between $a \parallel b$ and $a.b + b.a$





Interleaving

Concurrency

Expressiveness power of Higher Dimensional Automata

R.J. van Glabbeek / Electronic Notes in Theoretical Computer Science 128 (2005) 5–34

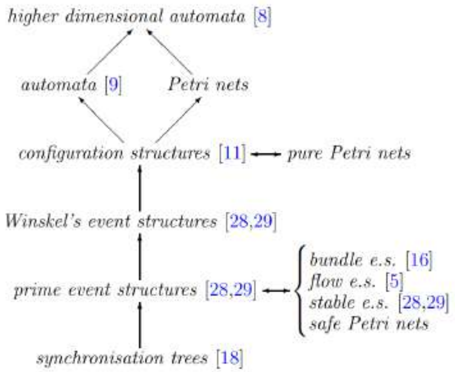


Fig. 1. A hierarchy of concurrency models, ordered by expressive power up to history preserving bisimulation

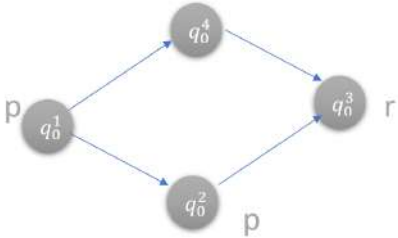
Modal logic and Higher Dimensional Modal Logic

Modal Logic

- Basic modal language:

$$\phi := \psi \mid \bot \mid \phi \to \psi \mid \langle\,\rangle\phi$$

- Valuation $\mathcal{V}$ from states to propositions
  $\mathcal{V}(q_0^1) = \{p\}, \mathcal{V}(q_0^2) = \{p\}, \mathcal{V}(q_0^3) = \{r\}$
- standard modal operators
  $q_0^1 \models \langle\,\rangle p$ *or* $q_0^1 \models [\,]\langle\,\rangle r$ *but* $q_0^1 \not\models [\,]p$



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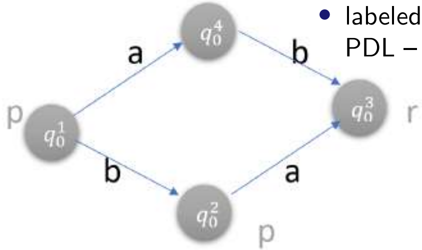

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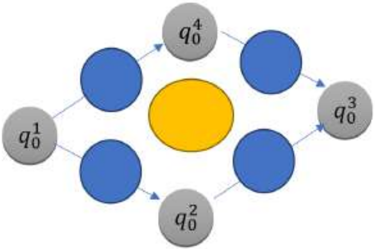
- standard modal operators $q_0^1 \models \langle\,\rangle p$ *or* $q_0^1 \models [\,]\langle\,\rangle r$ *but* $q_0^1 \not\models [\,]p$
- labeled operators (i.e., multi-modal logics, e.g., PDL – Dynamic Logic) $q_0^1 \models \langle a\rangle p$ *but* $q_0^1 \not\models \langle b\rangle p$

Higher Dimensional Modal Logic (HDML)

- Language of HDML

$$\phi := \psi \mid \perp \mid \phi \rightarrow \psi \mid \langle| \varphi \mid |\rangle \varphi$$



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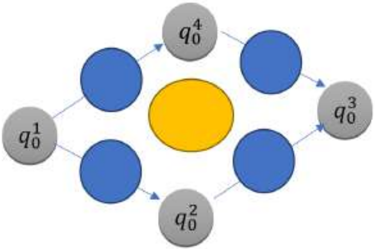

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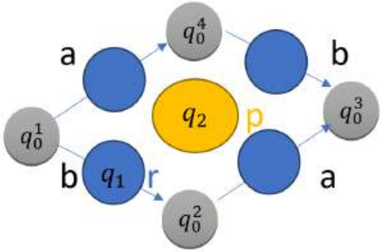

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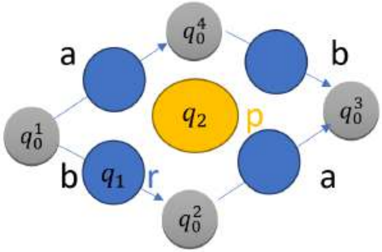
  During modality $\langle|$ *and* $[|\phi := \neg\langle|\neg\phi$



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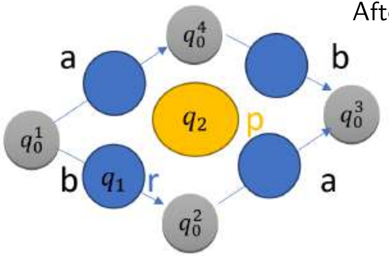
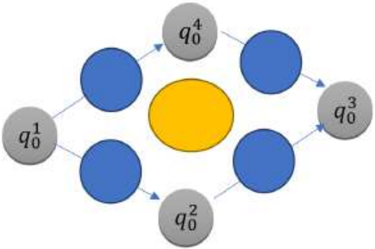


Forward modalities

- During modality $q \models \langle|\varphi, q \in Q_n$ iff

  $\exists q' \in Q_{n+1},\ s_i(q') = q, 1 \leq i \leq n{+}1 \text{ and } q' \models \phi$



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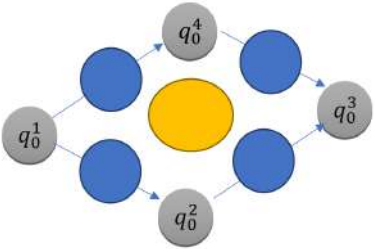
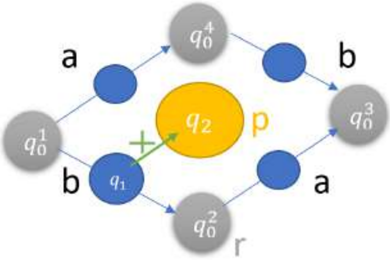
- After modality $q \models |\rangle\varphi, q \in Q_n$ iff

$$\exists q' \in Q_{n-1},\ t_i(q) = q', 1 \leq i \leq n \ and \ q' \models \phi$$



Forward modalities

- "Start a"



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- After modality $q \models | \rangle \varphi, q \in Q_n$ iff

  $\exists q' \in Q_{n-1},\ t_i(q) = q', 1 \leq i \leq n$ *and* $q' \models \phi$

- $q_1 \models \langle a | p, \quad s_2(q_2) = q_1$

Forward modalities

- "Start a"
- "terminate a"

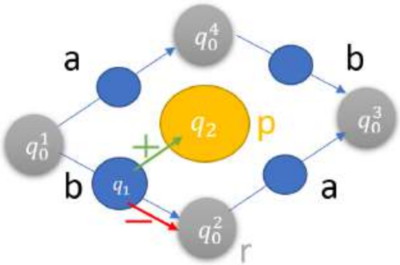
- During modality $q \models \langle|\varphi, q \in Q_n$ iff

  $\exists q' \in Q_{n+1},\ s_i(q') = q, 1 \leq i \leq n+1 \ and \ q' \models \phi$

- After modality $q \models |\rangle\varphi, q \in Q_n$ iff

  $\exists q' \in Q_{n-1},\ t_i(q) = q', 1 \leq i \leq n \ and \ q' \models \phi$

- $q_1 \models \langle a|p \wedge |b\rangle r, \quad s_2(q_2) = q_1, \quad t_1(q_1) = q_0^2$



A path in a Higher Dimensional Automata

[van Glabbeek On the expressiveness of Higher Dimensional Automata]

Definition 17. A *path* in an HDA (Q, s, t, I, F, l) is a sequence of pairs $(\partial_1, q_1)(\partial_2, q_2)\dots(\partial_m, q_m)$, denoted $I \xrightarrow{\partial_1} q_1 \xrightarrow{\partial_2} q_2 \xrightarrow{\partial_3} \dots \xrightarrow{\partial_m} q_m$, with $q_k \in Q$ and $\partial_k \in \{s_i, t_i \mid 1 \leqslant i\}$ for $1 \leqslant k \leqslant m$, such that

$$q_{k-1} = s_i(q_k) \quad \text{if } \partial_k = s_i \quad \text{and} \quad q_k = t_i(q_{k-1}) \text{ if } \partial_k = t_i.$$

Here $q_0 := I$, i.e. I consider only paths starting from the initial state. One writes $end(\pi)$ for q_m.

Logical characterizations of bisimulations

Modal characterizations of bisimulations

Definition

- Two pointed models are modally equivalent $(\mathcal{H}, \mathcal{V}, q) \leftrightsquigarrow (\mathcal{H}', \mathcal{V}', q')$ if:

$$\forall \varphi. \;\; \mathcal{H}, \mathcal{V}, q \models \varphi \;\; \textit{iff} \;\; \mathcal{H}', \mathcal{V}', q' \models \varphi$$

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- Two paths $\pi \in \mathcal{H}$ and $\pi' \in \mathcal{H}'$ are called modally equivalent if

$$\textit{length}(\pi) = \textit{length}(\pi') = m \ \textit{and} \ (\mathcal{H}, \mathcal{V}, q_i) \leftrightsquigarrow (\mathcal{H}', \mathcal{V}', q_i'), \forall 1 \leq i \leq m$$

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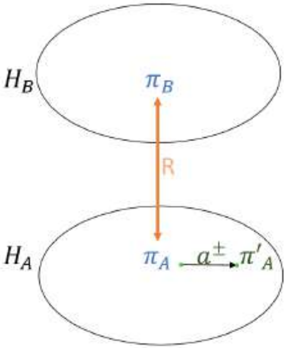
- A bisimulation $\approx$ is characterized by a modal logic when

$$\mathcal{H}, \mathcal{V}, q \approx \mathcal{H}', \mathcal{V}', q' \ \ \mathit{iff} \ \ \mathcal{H}, \mathcal{V}, q \leftrightsquigarrow \mathcal{H}', \mathcal{V}', q'$$

Split-bisimualtion

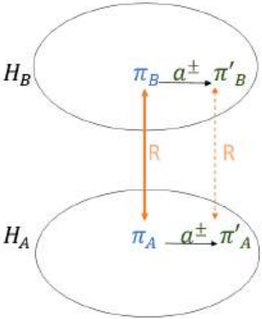


Two higher dimensional automata $(\mathcal{H}_A, q^0_A)$ and $(\mathcal{H}_B, q^0_B)$ (with q^0_A and q^0_B are inital cells) are called split bisimulation equivalent ($\mathcal{H}_B \approx_{split} \mathcal{H}_B$) if there exists a binary relation R between their paths starting from q^0_A (respectively q^0_B) satisfies the following:

0. *If* $\pi_A R \pi_B$ *then* π_A *and* π_B *have the same length and each correspondent cells satisfy the same propositional letters.*

1. *if* $\pi_A R \pi_B$ *and* $\pi_A \xrightarrow{a^\pm} \pi'_A$ *then* $\exists \pi'_B$ *with* $\pi_B \xrightarrow{a^\pm} \pi'_B$ *and* $\pi'_A R \pi'_B$*;*
2. *if* $\pi_A R \pi_B$ *and* $\pi_B \xrightarrow{a^\pm} \pi'_B$ *then* $\exists \pi'_A$ *with* $\pi_A \xrightarrow{a^\pm} \pi'_A$ *and* $\pi'_A R \pi'_B$*;*

Split-bisimualtion



Two higher dimensional automata $(\mathcal{H}_A, q_A^0)$ and $(\mathcal{H}_B, q_B^0)$ (with q_A^0 and q_B^0 are inital cells) are called split bisimulation equivalent ($\mathcal{H}_B \approx_{split} \mathcal{H}_B$) if there exists a binary relation R between their paths starting from q_A^0 (respectively q_B^0) satisfies the following:

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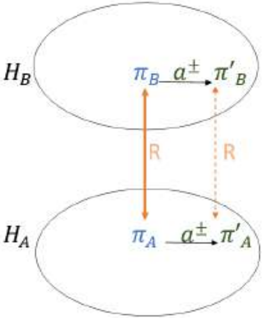
$$\pi_A \xrightarrow{a^+} \pi'_A$$

$$\pi_A: \; I \xrightarrow{\partial_1} q_1 \xrightarrow{\partial_2} q_2 \xrightarrow{\partial_3} \dots \xrightarrow{\partial_m} q_m$$

$$\pi'_A: \; I \xrightarrow{\partial_1} q_1 \xrightarrow{\partial_2} q_2 \xrightarrow{\partial_3} \dots \xrightarrow{\partial_m} q_m \xrightarrow{s_i} q_{m+1}$$

Split-bisimualtion



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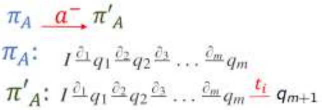

Logical Characterization of split-bisimulation

Theorem

HDML characterizes split-bisimulation.

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- $\approx_{HDML} \quad \subset \quad \approx_{split-bis}$

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  $\approx_{HDML}$ satisfies:

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  2. if $\pi_A R \pi_B$ and $\pi_B \xrightarrow{a^{\pm}} \pi'_B$ then $\exists \pi'_A$ with $\pi_A \xrightarrow{a^{\pm}} \pi'_A$ and $\pi'_A R \pi'_B$;

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- $\approx_{HDML} \quad \supset \quad \approx_{split-bis}$ *reductio ad absurdum*

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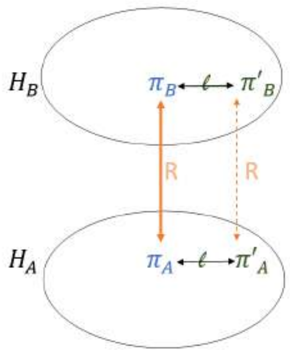
- $\approx_{HDML} \quad \supset \quad \approx_{split-bis}$ *reductio ad absurdum*
  Structural induction on formula. Forward modalities
  1. a^+ During modality "$\langle a|$"
  2. a^- After modality "$|a\rangle$"

History preserving bisimulation



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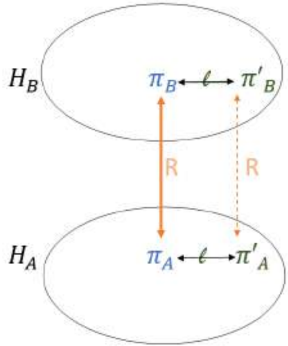
3. if $\pi_A R \pi_B$ and $\pi_A \xleftrightarrow{l} \pi'_A$ then $\exists \pi'_B$ with $\pi_B \xleftrightarrow{l} \pi'_B$ and $\pi'_A R \pi'_B$;
4. if $\pi_A R \pi_B$ and $\pi_B \xleftrightarrow{l} \pi'_B$ then $\exists \pi'_A$ with $\pi_A \xleftrightarrow{l} \pi'_A$ and $\pi'_A R \pi'_B$;

History preserving bisimulation



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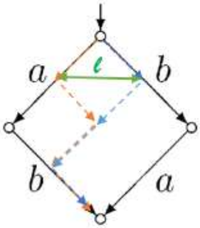
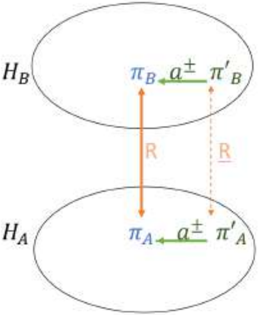


$a^+b^+a^-b^-$, $b^+a^+a^-b^-$

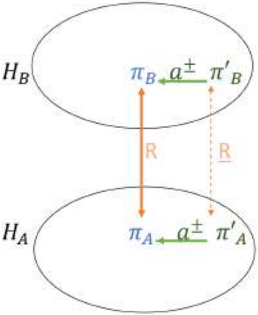
Hereditary history preserving bisimulation



1. *if* $\pi_A R \pi_B$ *and* $\pi_A \xrightarrow{a^{\pm}} \pi'_A$ *then* $\exists \pi'_B$ *with* $\pi_B \xrightarrow{a^{\pm}} \pi'_B$ *and* $\pi'_A R \pi'_B$;
2. *if* $\pi_A R \pi_B$ *and* $\pi_B \xrightarrow{a^{\pm}} \pi'_B$ *then* $\exists \pi'_A$ *with* $\pi_A \xrightarrow{a^{\pm}} \pi'_A$ *and* $\pi'_A R \pi'_B$;
3. *if* $\pi_A R \pi_B$ *and* $\pi_A \xleftrightarrow{l} \pi'_A$ *then* $\exists \pi'_B$ *with* $\pi_B \xleftrightarrow{l} \pi'_B$ *and* $\pi'_A R \pi'_B$;
4. *if* $\pi_A R \pi_B$ *and* $\pi_B \xleftrightarrow{l} \pi'_B$ *then* $\exists \pi'_A$ *with* $\pi_A \xleftrightarrow{l} \pi'_A$ *and* $\pi'_A R \pi'_B$;
5. *if* $\pi_A R \pi_B$ *and* $\pi'_A \xrightarrow{a^{\pm}} \pi_A$ *then* $\exists \pi'_B$ *with* $\pi'_B \xrightarrow{a^{\pm}} \pi_B$ *and* $\pi'_A R \pi'_B$;
6. *if* $\pi_A R \pi_B$ *and* $\pi'_B \xrightarrow{a^{\pm}} \pi_B$ *then* $\exists \pi'_A$ *with* $\pi'_A \xrightarrow{a^{\pm}} \pi_A$ *and* $\pi'_A R \pi'_B$.

Denote this as $(\mathcal{H}_A, q_A^0) \overset{hh}{\sim} (\mathcal{H}_B, q_B^0)$.

Hereditary history preserving bisimulation



1. if $\pi_A R \pi_B$ and $\pi_A \xrightarrow{a^\pm} \pi'_A$ then $\exists \pi'_B$ with $\pi_B \xrightarrow{a^\pm} \pi'_B$ and $\pi'_A R \pi'_B$;
2. if $\pi_A R \pi_B$ and $\pi_B \xrightarrow{a^\pm} \pi'_B$ then $\exists \pi'_A$ with $\pi_A \xrightarrow{a^\pm} \pi'_A$ and $\pi'_A R \pi'_B$;
3. if $\pi_A R \pi_B$ and $\pi_A \overset{l}{\longleftrightarrow} \pi'_A$ then $\exists \pi'_B$ with $\pi_B \overset{l}{\longleftrightarrow} \pi'_B$ and $\pi'_A R \pi'_B$;
4. if $\pi_A R \pi_B$ and $\pi_B \overset{l}{\longleftrightarrow} \pi'_B$ then $\exists \pi'_A$ with $\pi_A \overset{l}{\longleftrightarrow} \pi'_A$ and $\pi'_A R \pi'_B$;
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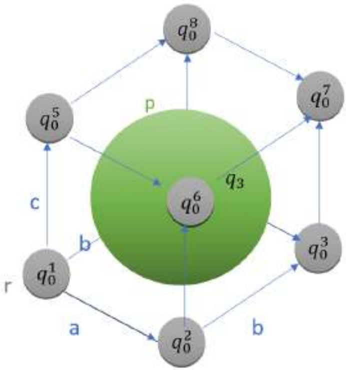
Backward modality??

h-HDML

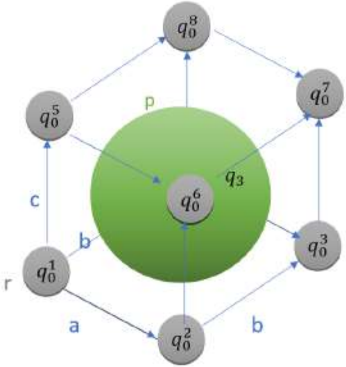


- Language of h-HDML

$$\phi := \psi \mid \bot \mid \phi \rightarrow \psi \mid \langle\,\rangle\phi \mid |\rangle\varphi \mid \langle|\varphi \mid \overleftarrow{|\rangle}\varphi \mid \overleftarrow{\langle|}\varphi$$

- Valuation $\mathcal{V}$ from cells to propositions.
- labeled operators $|a\rangle$, $\langle a|$, $\overleftarrow{|a\rangle}$ and $\overleftarrow{\langle b|}$
  $q_0^1 \models \langle a|\langle b|\langle c|p$ and $q_0^7 \models \overleftarrow{|a\rangle}\overleftarrow{|b\rangle}\overleftarrow{|c\rangle}\overleftarrow{\langle a|}\overleftarrow{\langle b|}\overleftarrow{\langle c|}r$

h-HDML



- Language of h-HDML

$$\phi := \psi \mid \bot \mid \phi \to \psi \mid \langle\,\rangle\phi \mid |\rangle\varphi \mid \langle|\varphi \mid \overleftarrow{|\rangle}\,\varphi \mid \overleftarrow{\langle|}\,\varphi$$

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Logical characterization of hh- bisimulation

Conjecture

h-HDML characterizes hereditary history preseving-bisimulation.

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  Structural induction on formula.

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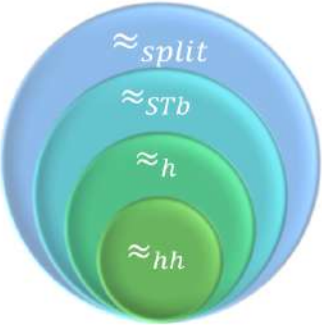
- $\approx_{h-HDML} \quad \supset \quad \approx_{hh-bis}$ reductio ad absurdum
  Structural induction on formula.
  - 1,2 for forward modalities.
  - 5,6 for backward modalities.

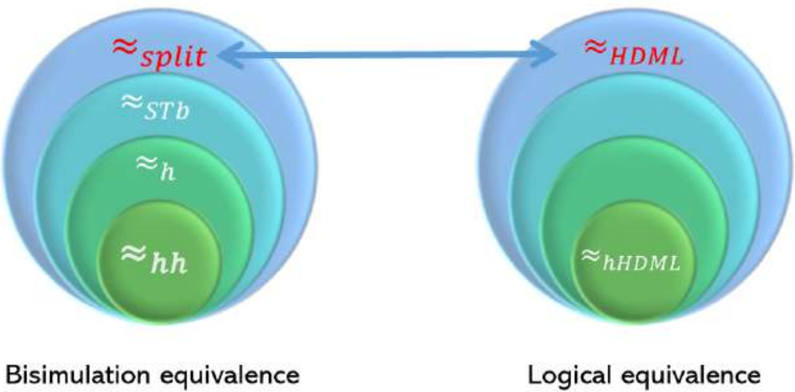
Summary and future work

Concurrent bisimulation

[van Glabbeek & Vaandrager 1997]

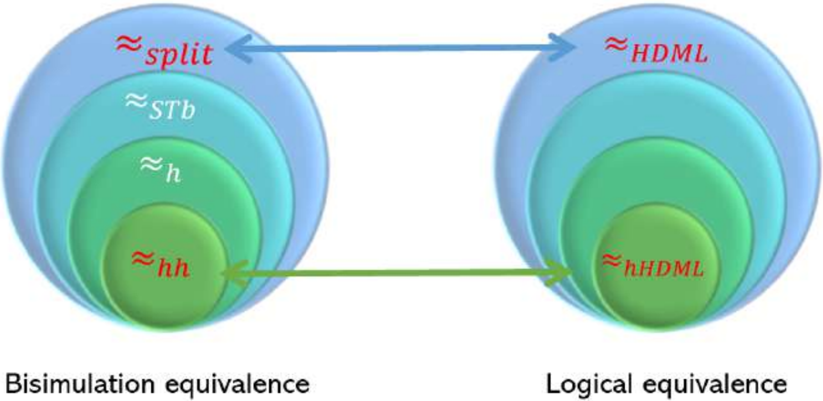


Sum up



≈split
≈STb
≈h
≈hh
≈HDML
≈hHDML
Bisimulation equivalence
Logical equivalence

Sum up


$\approx_{split}$
$\approx_{STb}$
$\approx_h$
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$\approx_{HDML}$
$\approx_{hHDML}$
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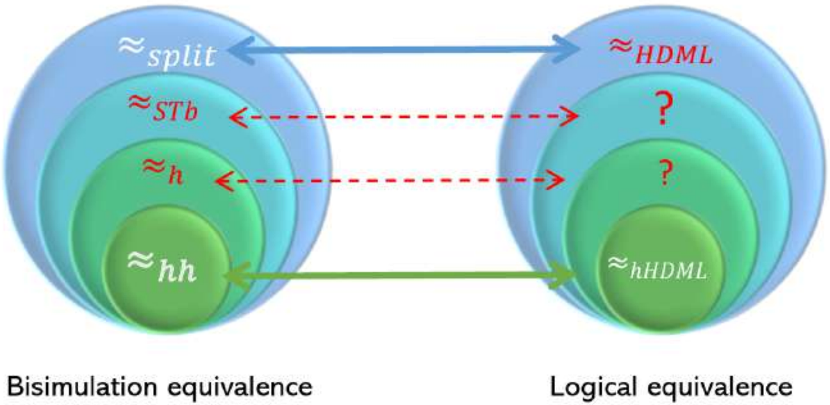
Future Work



≈split
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?
?
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Thank you!