Orbit-finite systems of linear equations

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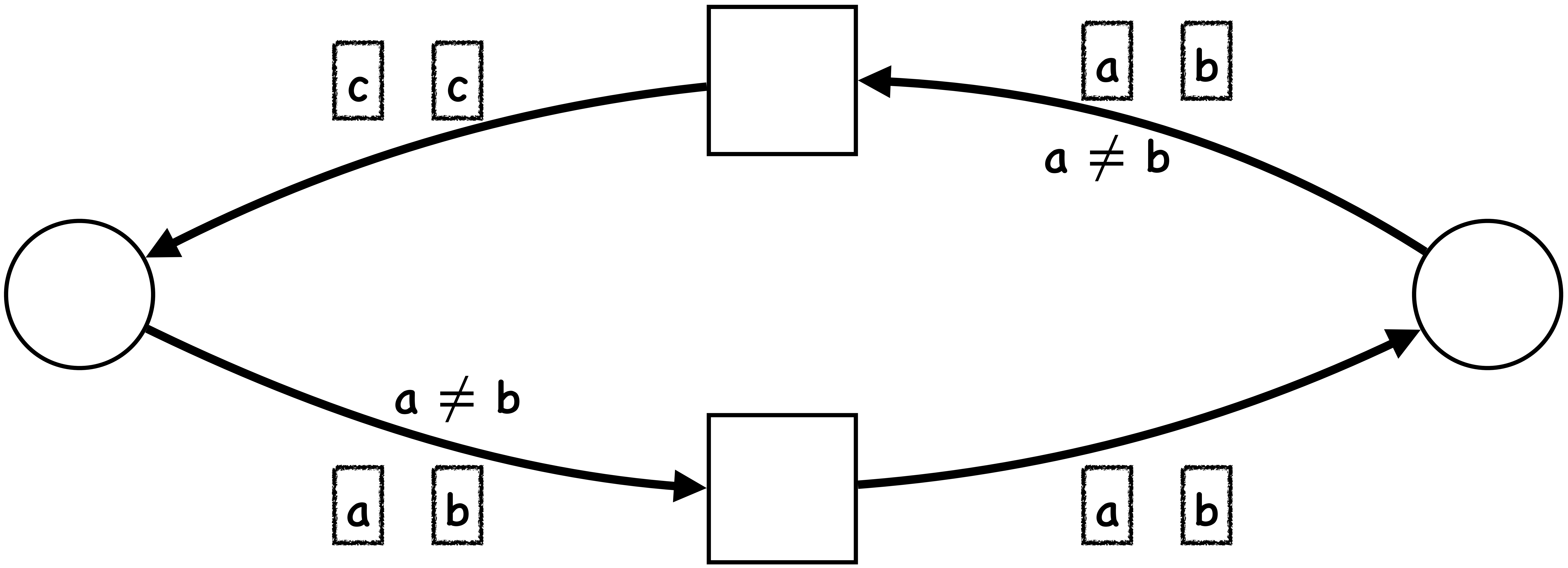
Joint work with Piotr Hofman and Sławomir Lasota

Petri nets with coloured/labelled tokens

Example



Petri nets with coloured/labelled tokens

How to solve reachability/coverability/… for Petri nets with labelled tokens?

Finitely many labels

⇓

Petri net with places P and labels L

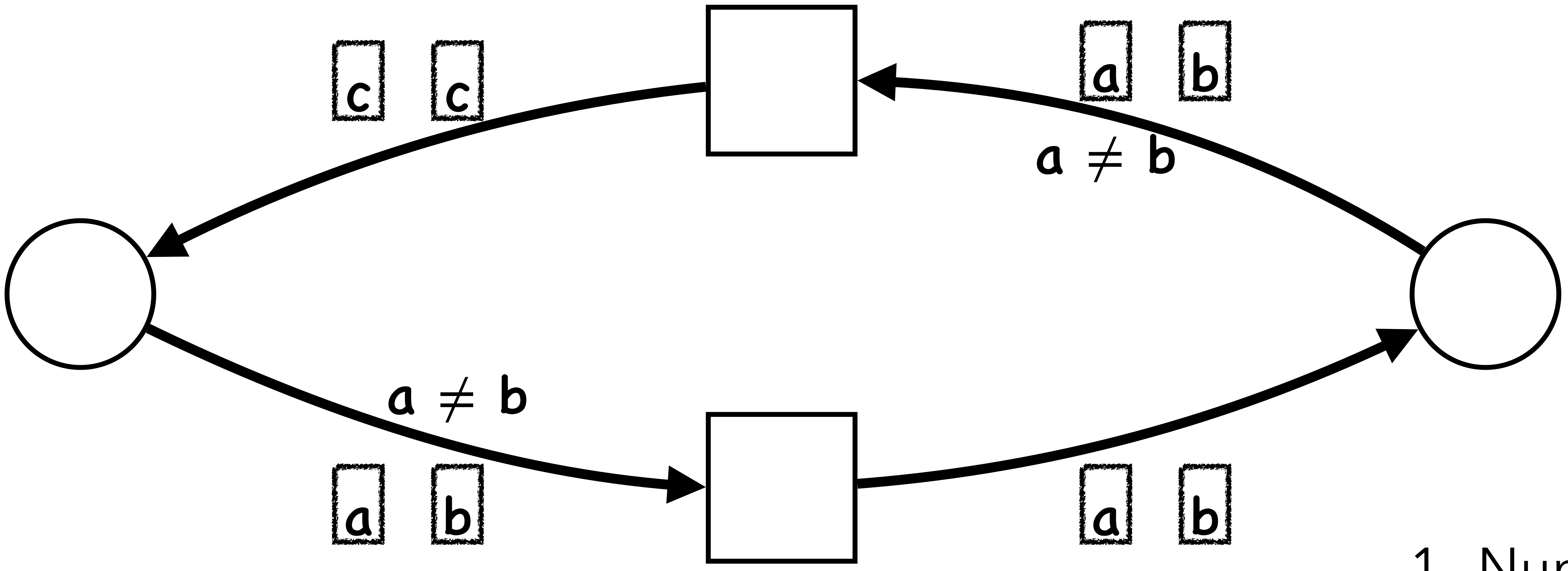
$\mapsto$

Petri net with places $(P \times L)$

1. Complexity depends on the number of labels

2. Does not work if number of labels are (potentially) infinite

Petri nets with coloured/labelled tokens



1. Number of labels can be infinite.
2. Transitions are symmetric with respect to the labels.

Petri nets with coloured/labelled tokens

Idea : Exploit the symmetry to give algorithms for reachability/coverability/… for Petri nets with labelled tokens.

Possible bonus : The same algorithm works for Petri nets with tokens labelled with finitely many (but large enough number of) labels. Complexity of this algorithm is independent of the number of labels!

Petri nets with ~~coloured/labelled tokens~~ data

Reachability (open)

Finding non-negative integer solution of systems of equations

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Integer reachability

Continuous reachability

1. Continuous Reachability for Unordered Data Petri nets is in PTime. Utkarsh Gupta, Preey Shah, S. Akshay, Piotr Hofman

2. Generalisation to orbit-finite setting (unpublished research)

1. Linear equations for unordered data vectors. Piotr Hofman, Jakub Różycki

2. Generalisation to orbit-finite setting follows from "Solvability of orbit-finite systems of linear equations Arka Ghosh, Piotr Hofman, Sławomir Lasota"

Orbit-finite systems of linear equations

Example

$$\mathbb{A}$$

An infinite set

$$\left\{ x_{(\alpha,\beta)} \;\mid\; \alpha \neq \beta \in \mathbb{A} \right\}$$

Set of variables

$$\left(x_{(\alpha,\beta)} + 2x_{(\beta,\alpha)} = 1 \right)_{\alpha \neq \beta \in \mathbb{A}}$$

System of equations

Orbit-finite systems of linear equations

Example

$$\pi \in \big(\text{Permutations of } \mathbb{A}\big)$$

$$x_{(\alpha,\beta)} \xrightarrow{\pi} x_{(\pi(\alpha),\pi(\beta))}$$

$$\left(x_{(\alpha,\beta)} + 2x_{(\beta,\alpha)} = 1\right) \xrightarrow{\pi} \left(x_{(\pi(\alpha),\pi(\beta))} + 2x_{(\pi(\beta),\pi(\alpha))} = 1\right)$$

Orbit-finite systems of linear equations

Example

$$\left(x_{(\alpha,\beta)} + 2x_{(\beta,\alpha)} = 1\right)_{\alpha\neq\beta\in\mathbb{A}}$$

$$\left(x_{(\alpha,\beta)} + 2x_{(\beta,\alpha)}\right) \xrightarrow{\pi} \left(x_{(\pi(\alpha),\pi(\beta))} + 2x_{(\pi(\beta),\pi(\alpha))}\right)$$

The systems of equations is invariant under the permutations of $\mathbb{A}$, and the equations are finitely many upto permutations of $\mathbb{A}$ (just one in this example).

Orbit-finite

Orbit-finite systems of linear equations

Decision problem(solv)
Input : An orbit-finite system of linear equations.
Question : Does it have a solution with a finite description?

Theorem : Solv is decidable, for integer and rational solutions.

(Theorem 4.4 in "A. Ghosh, P. Hofman, S. Lasota, Solvability of orbit-finite systems of linear equations. LICS'22")

$$x_{(\alpha,\beta)} = \frac{1}{3} \quad \forall \alpha \neq \beta \in \mathbb{A}$$

is a finitely describable solution for the system in the example

Orbit-finite systems of linear equations

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Question : What about inequalities?

(Results from unpublished research)

1. Undecidable for integer solutions.
2. Undecidable for finite integer solutions.
3. Decidable for rational solutions.
4. Decidable if every equation in the system is finitary.

Orbit-finite systems of linear equations

Orbit-finite basis theorem :

Let X be an orbit-finite set with atoms $\mathbb{A}$. The space of finitely supported linear functions from X has an orbit-finite basis.

$$\wr\wr$$

Let X be a first-order definable set with alphabet $\mathbb{A}$. The space of first-order definable linear functions from X has a first-order definable basis.