The complexity of soundness in workflow nets

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Joint work with
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UNIVERSITÉ DE SHERBROOKE

Processes are everywhere!

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  - Check applicant suitability
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Formally modeling processes: Workflow nets

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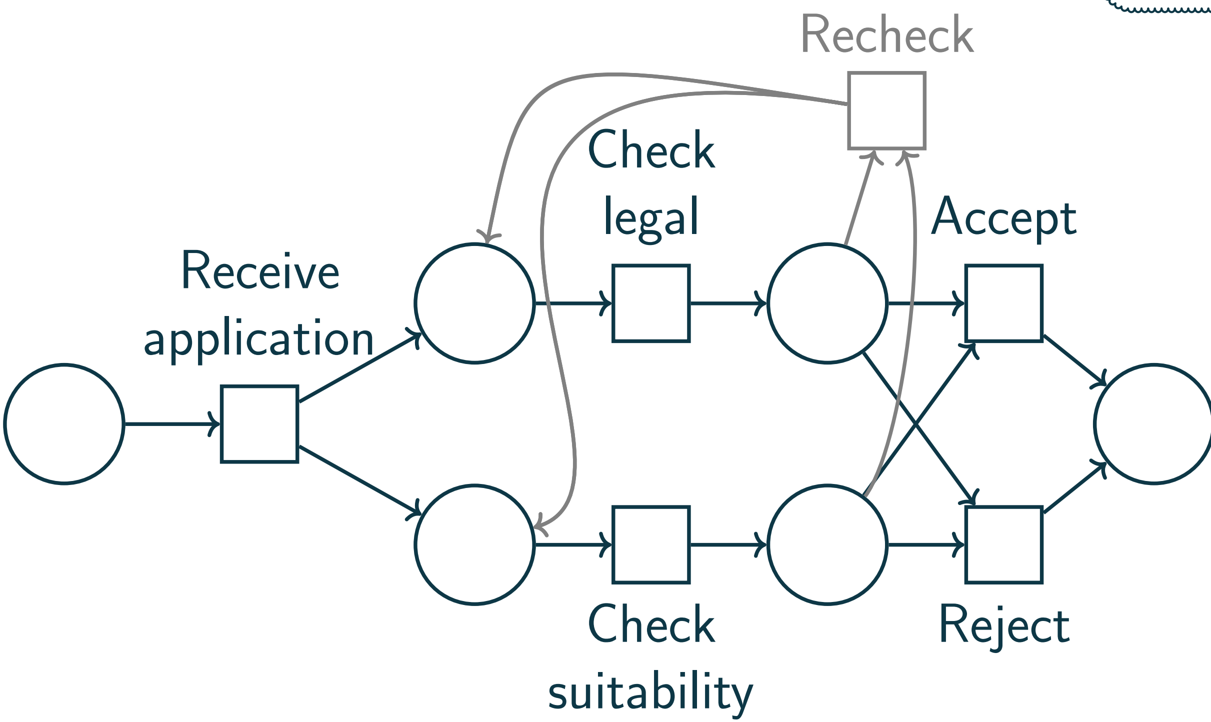

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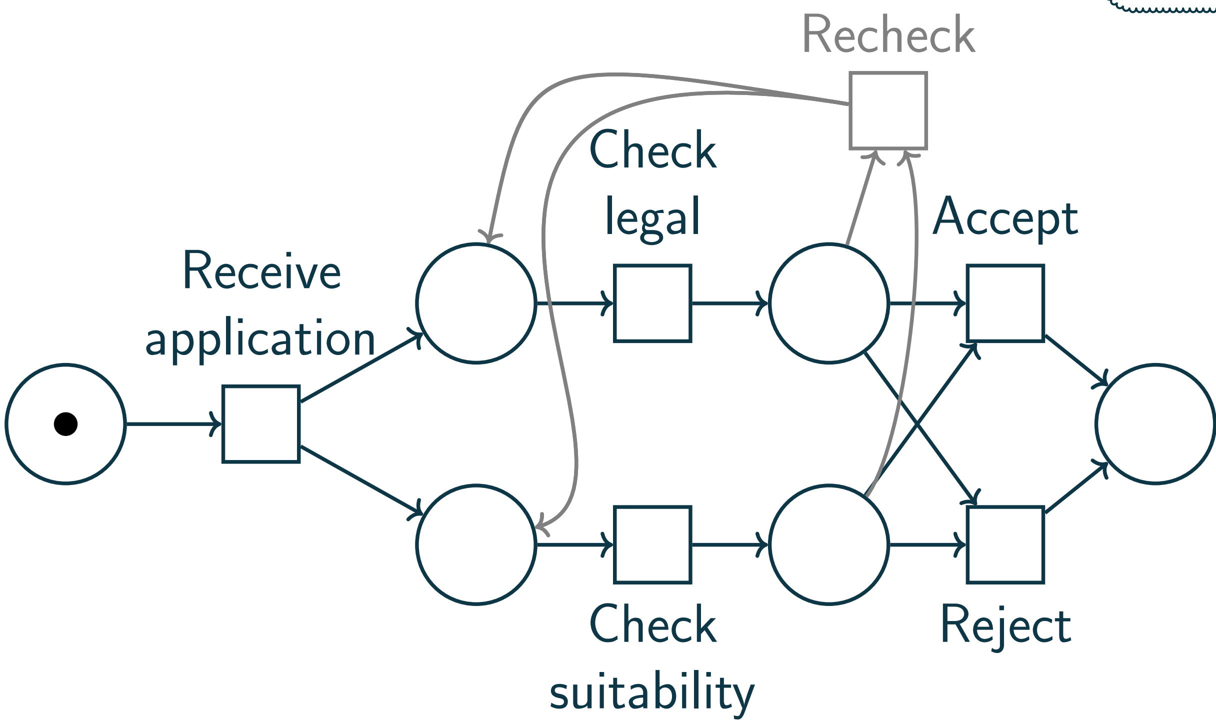

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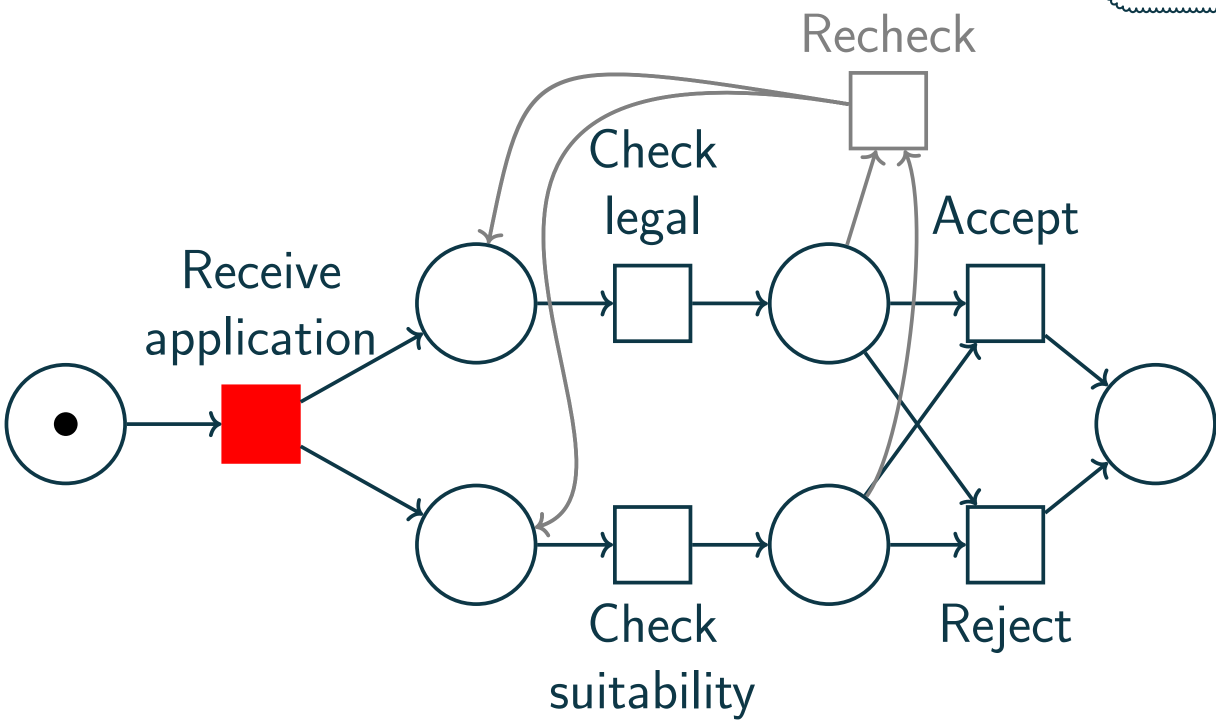

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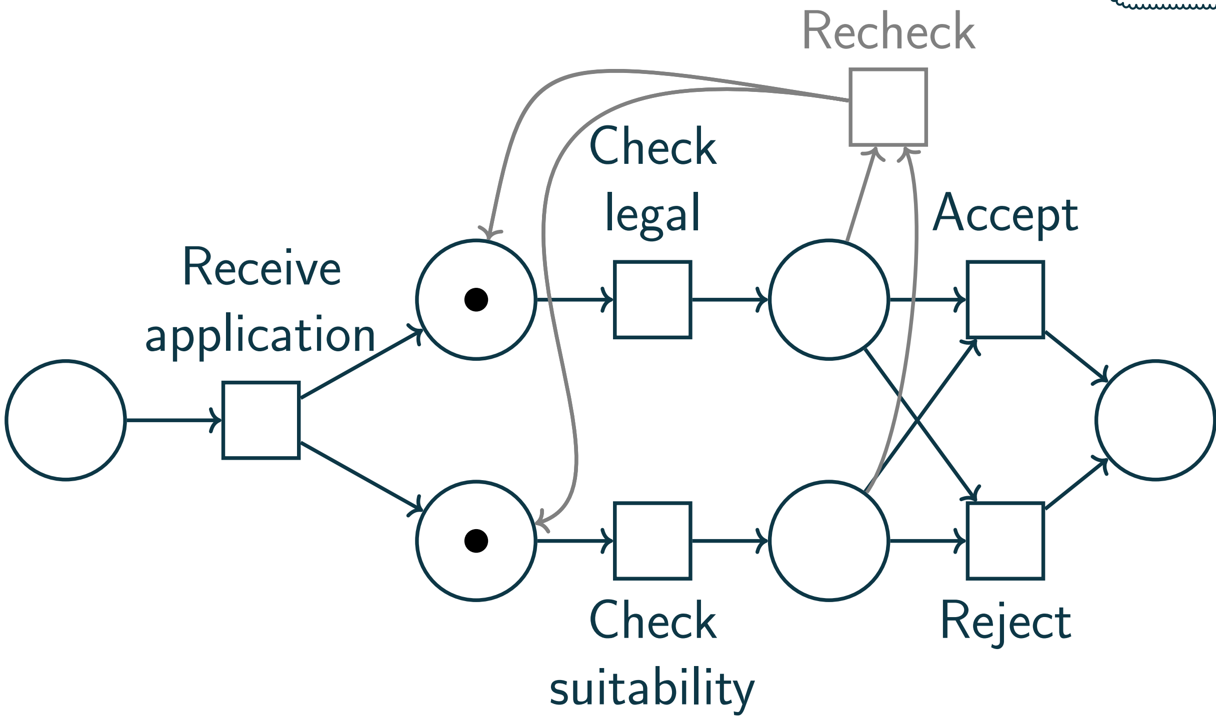
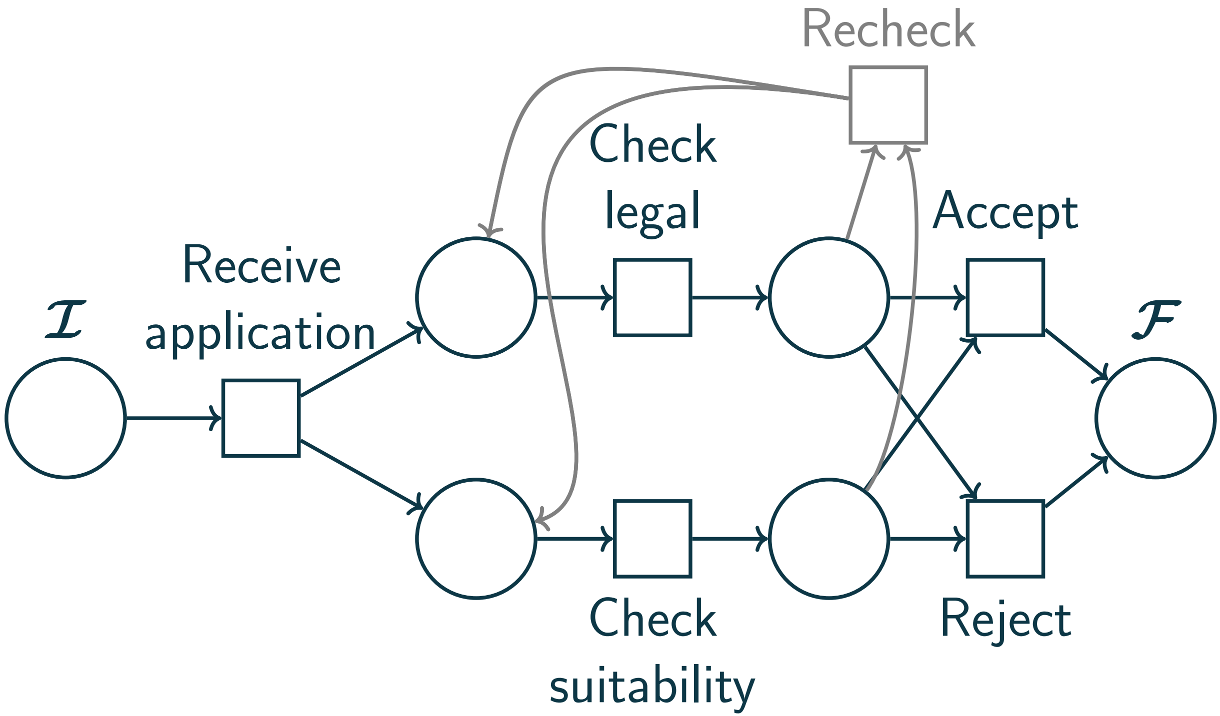


Correctness conditions for processes



Correctness conditions for processes

Option to complete:
We should be able to reach a
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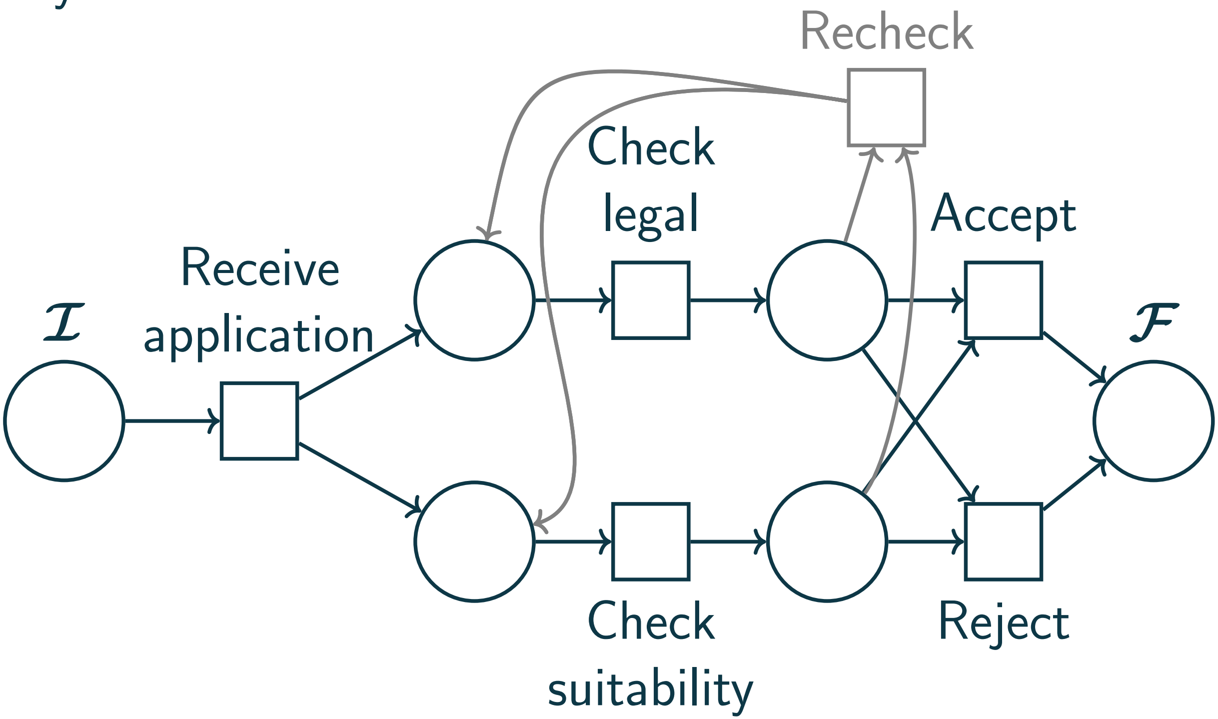

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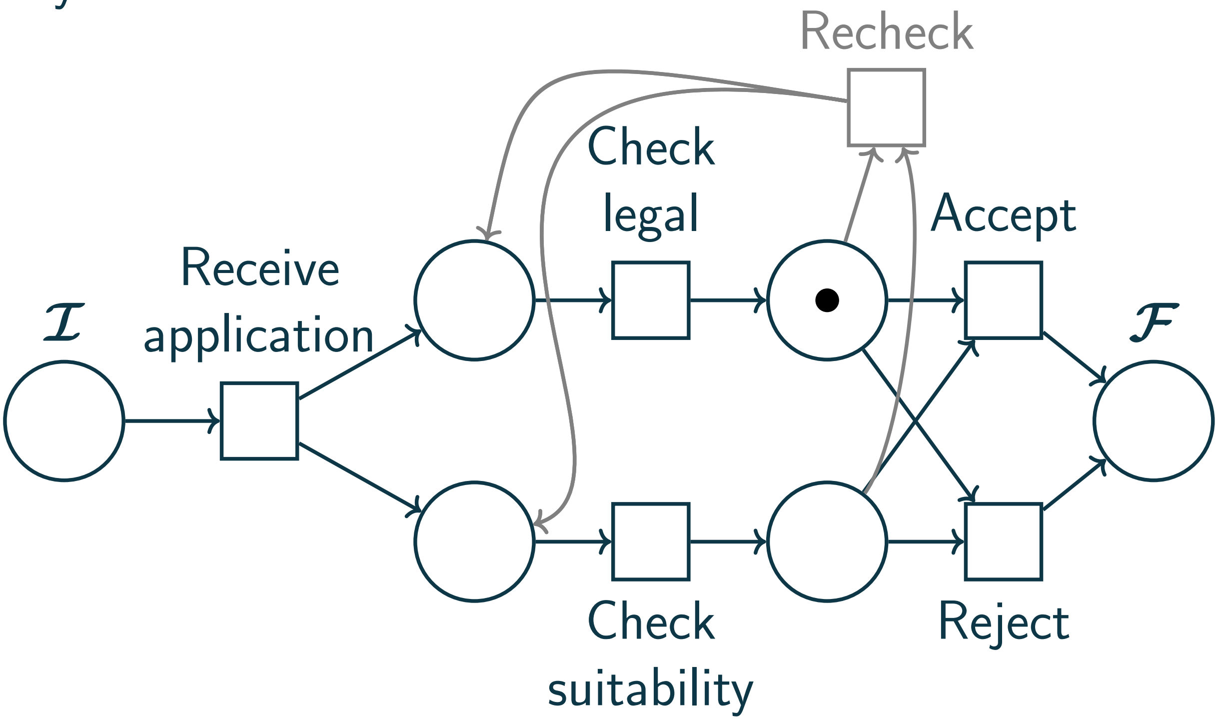

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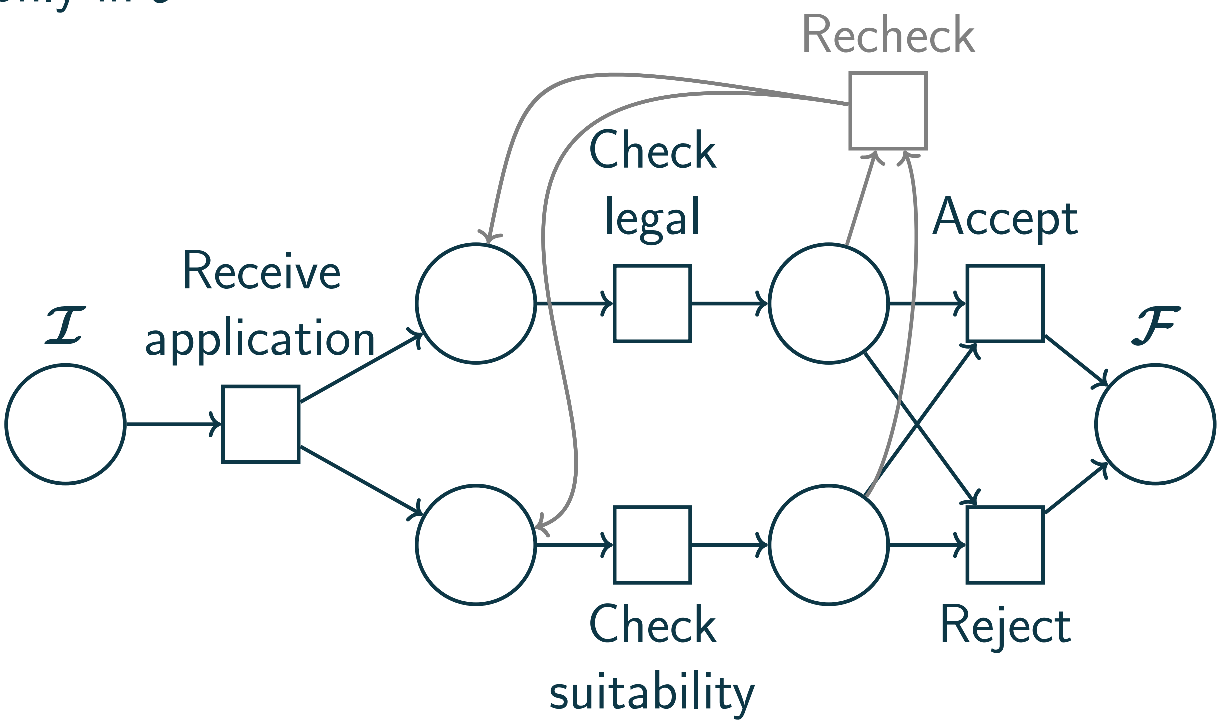

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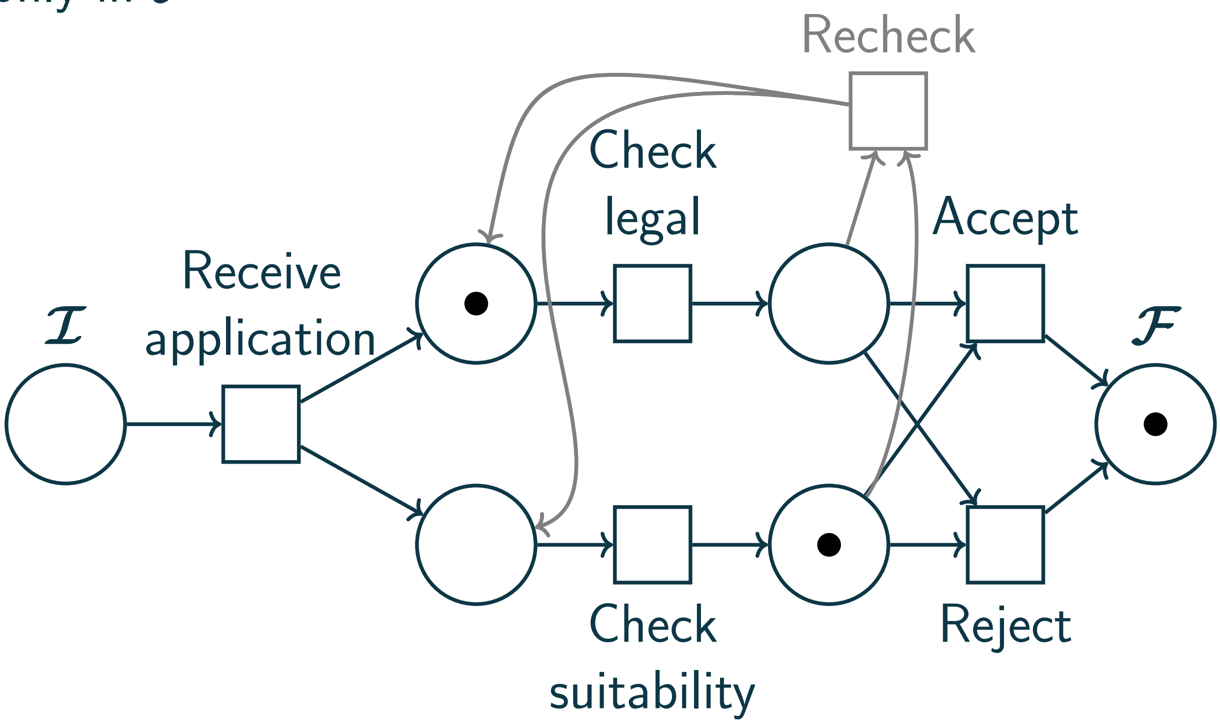

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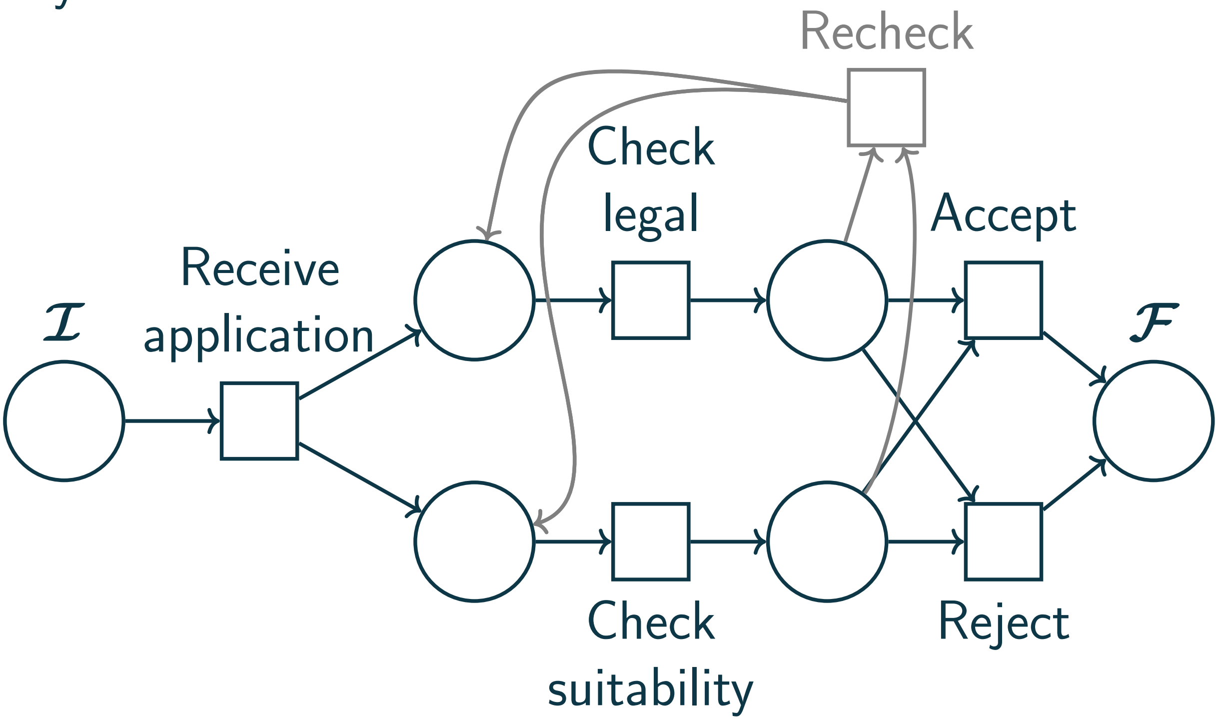
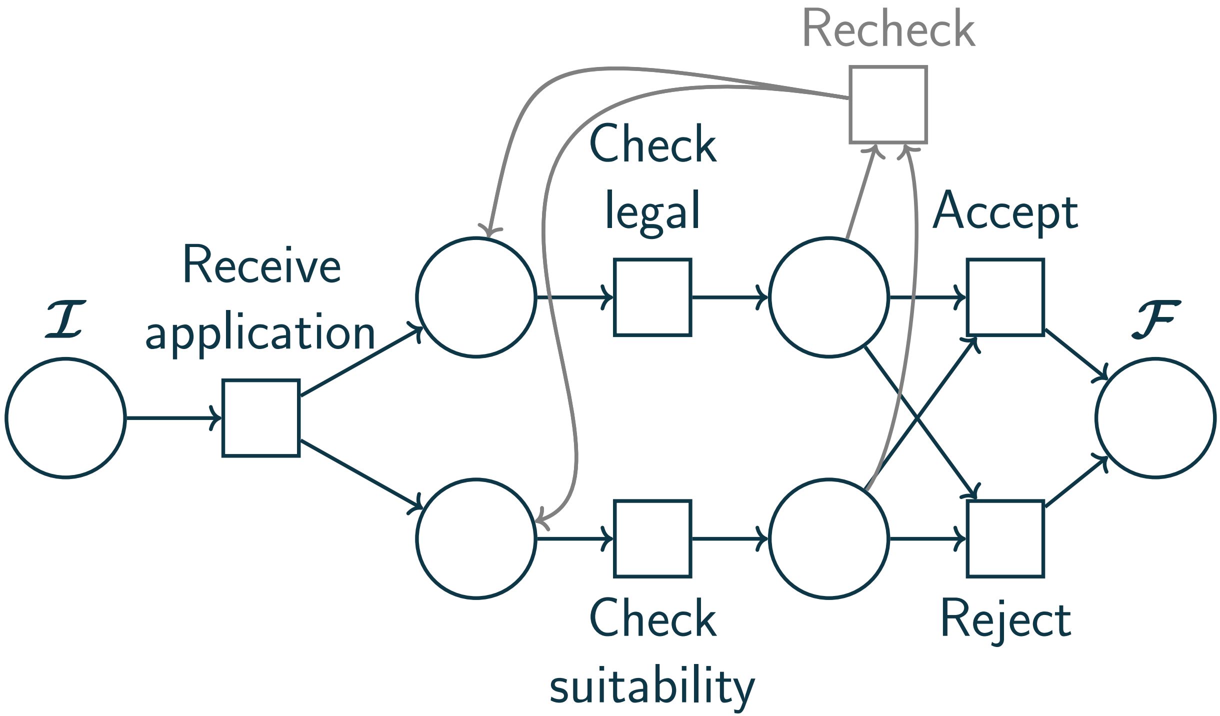


Can we condense these into a single condition?

A concise correctness condition

Soundness:
From any marking reachable from $\{\mathcal{I}\colon 1\}$, the final marking $\{\mathcal{F}\colon 1\}$ can be reached

$\forall$ runs π $\exists$ run π' : $\{\mathcal{I}\colon 1\} \xrightarrow{\pi\pi'} \{\mathcal{F}\colon 1\}$



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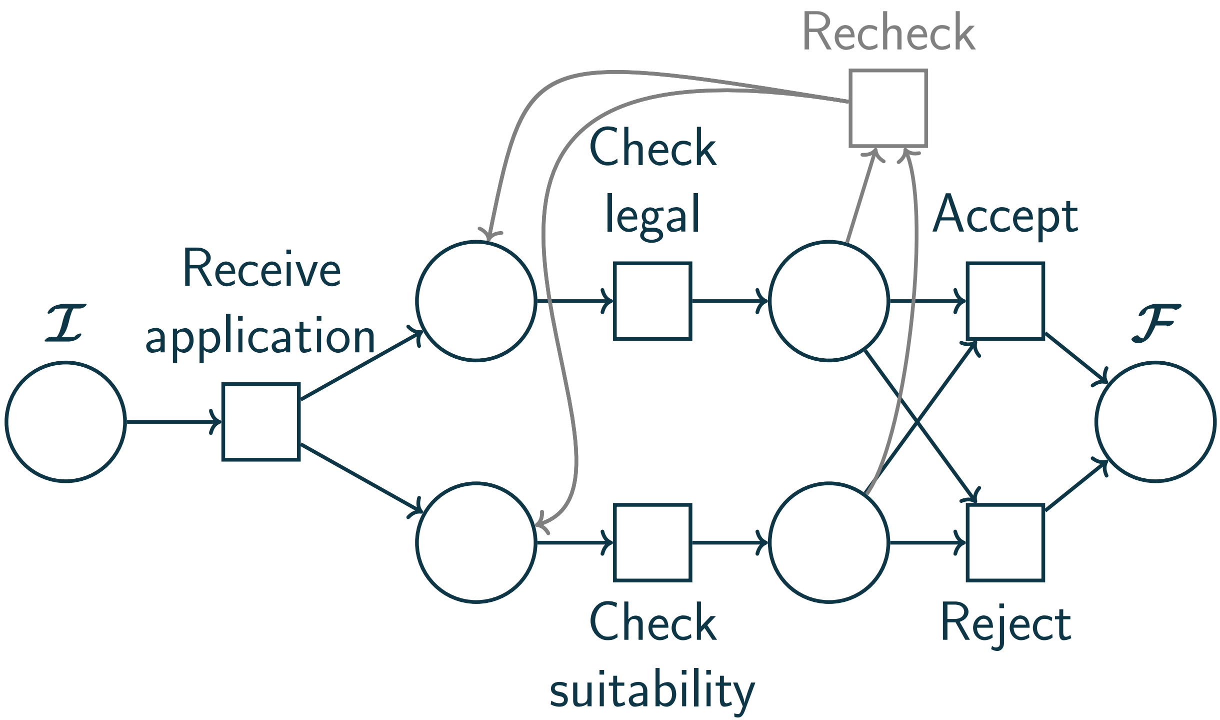

Extending soundness

***k*-soundness:**
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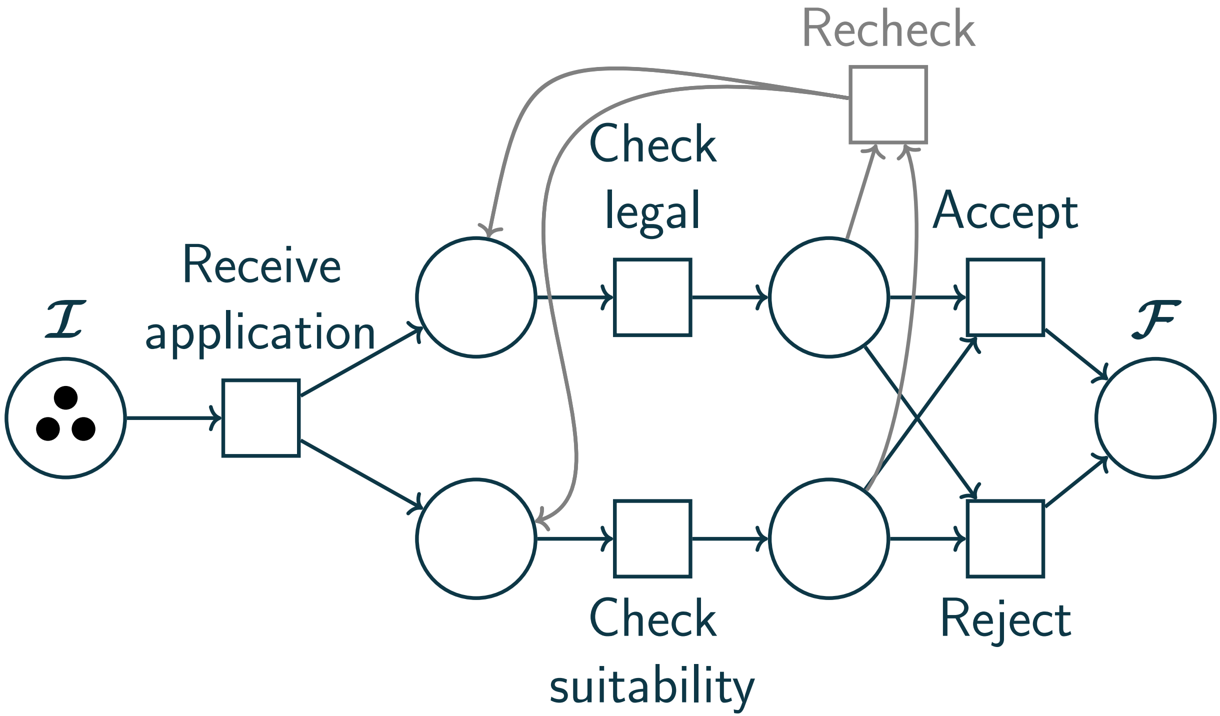

Variants of soundness

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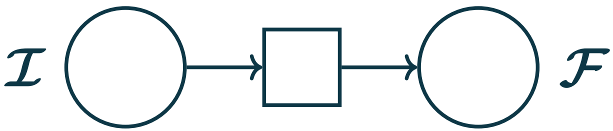

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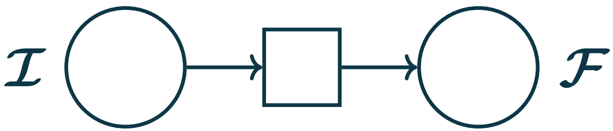	✓	

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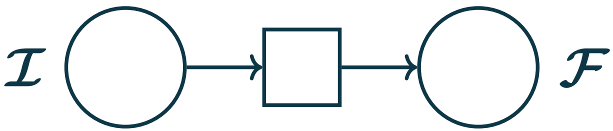	✓	✓

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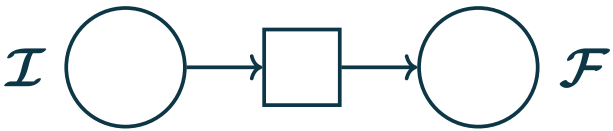
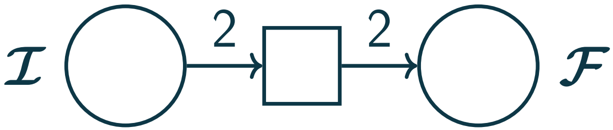
	✓	✓
		

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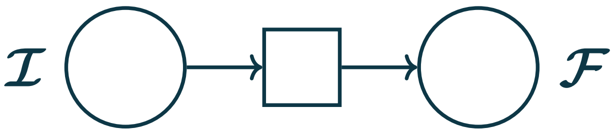
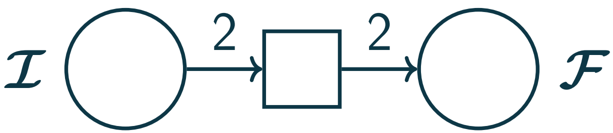
	✓	✓
	X Not 1-sound	

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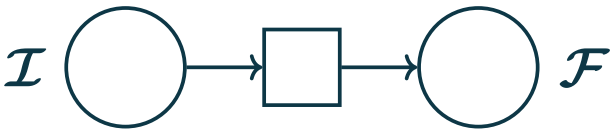
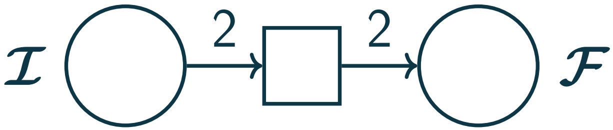
	✓	✓
	X Not 1-sound	✓ 2-sound

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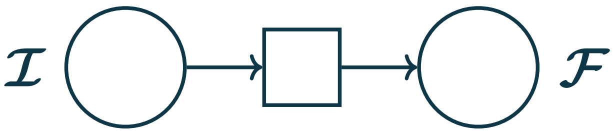
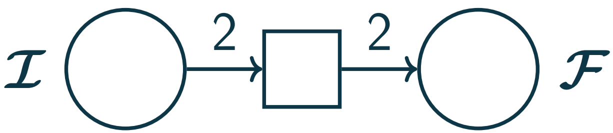
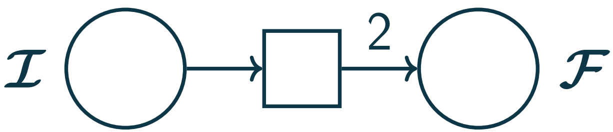
	✓	✓
	✗ Not 1-sound	✓ 2-sound
		

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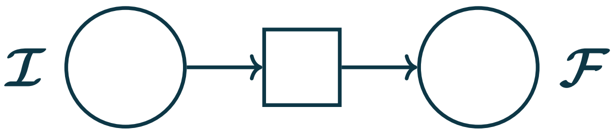
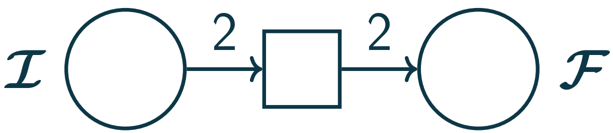
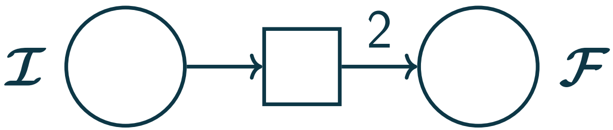
	✓	✓
	X Not 1-sound	✓ 2-sound
	X	

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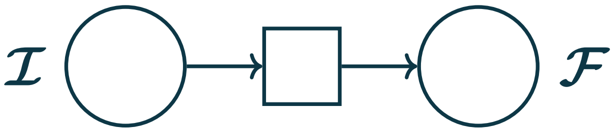
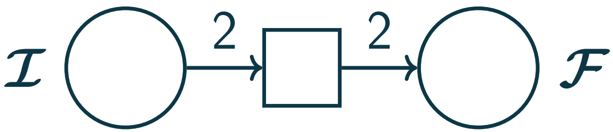
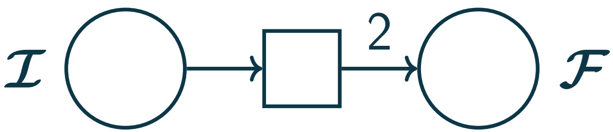
	✓	✓
	X Not 1-sound	✓ 2-sound
	X	X

Checking soundness - complexity?

	known results	our work
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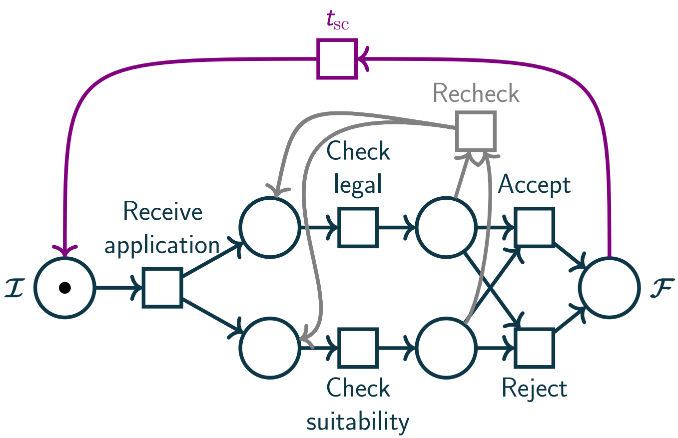

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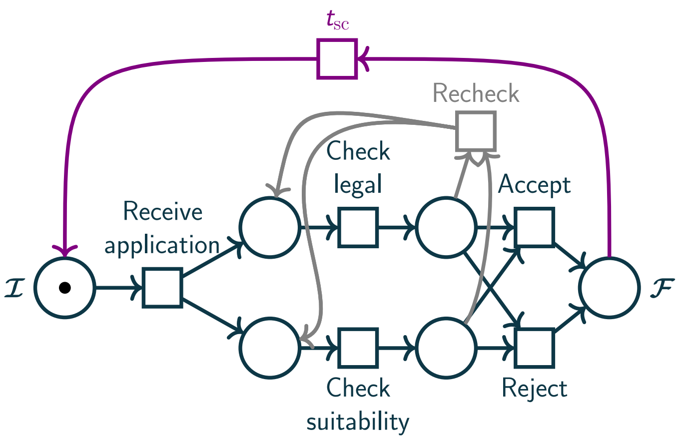

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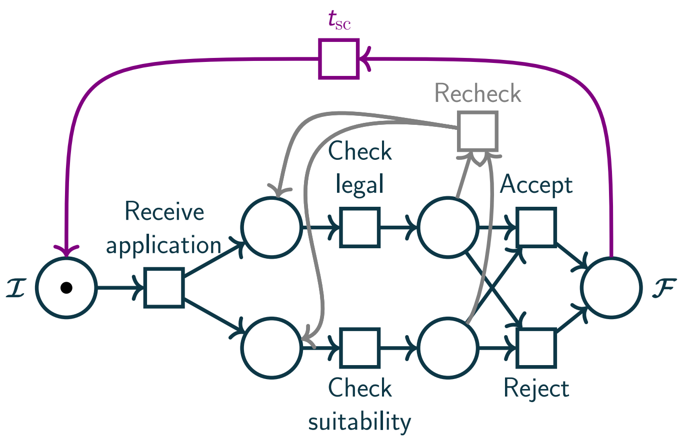

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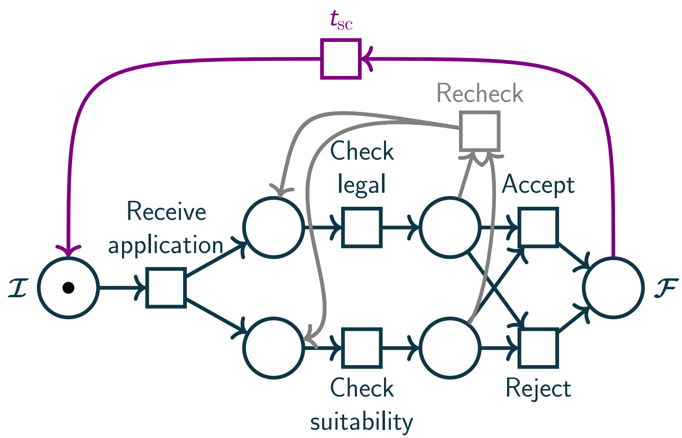

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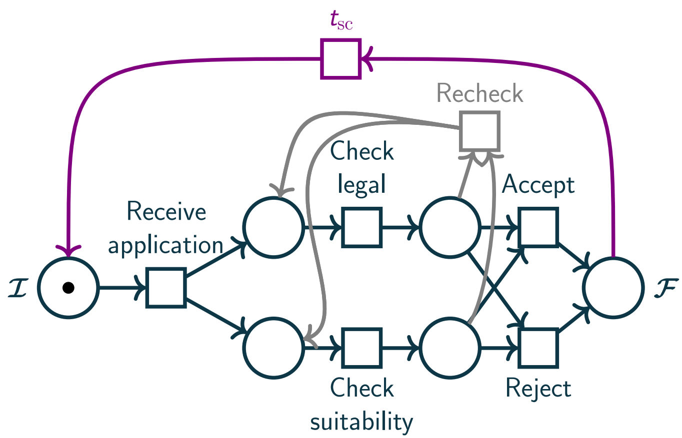

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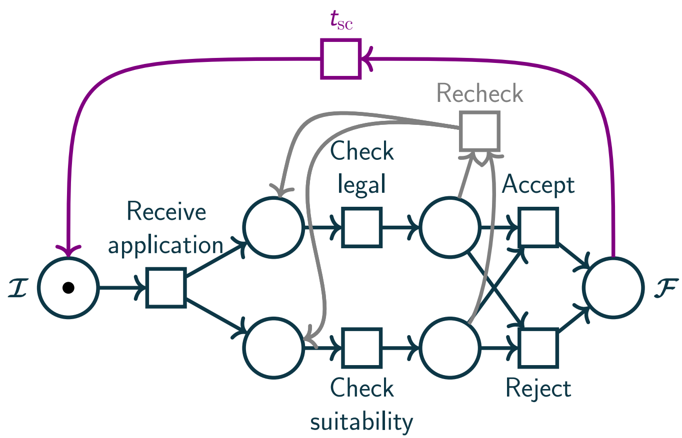

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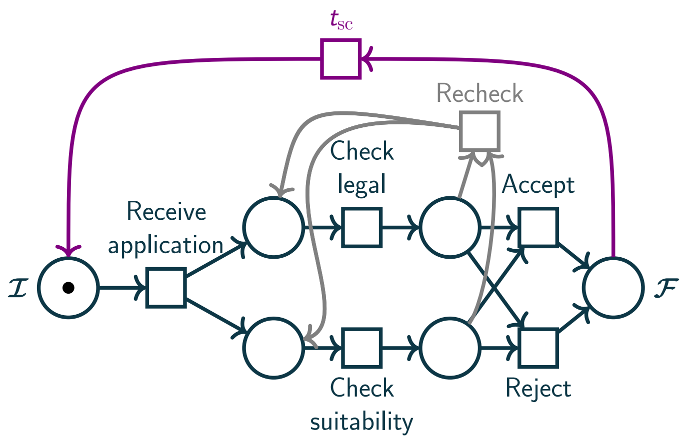

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$\downarrow$

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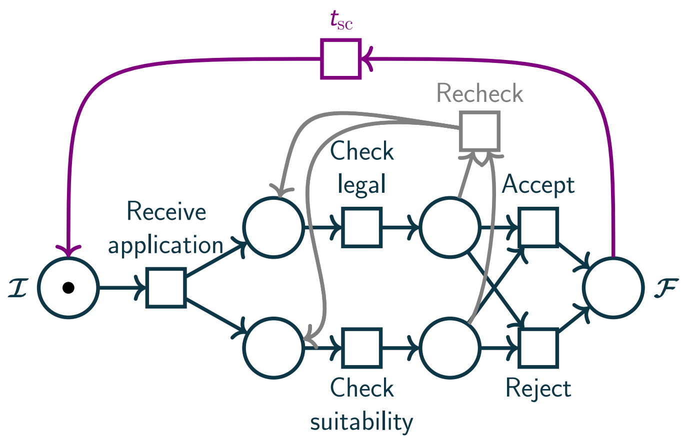

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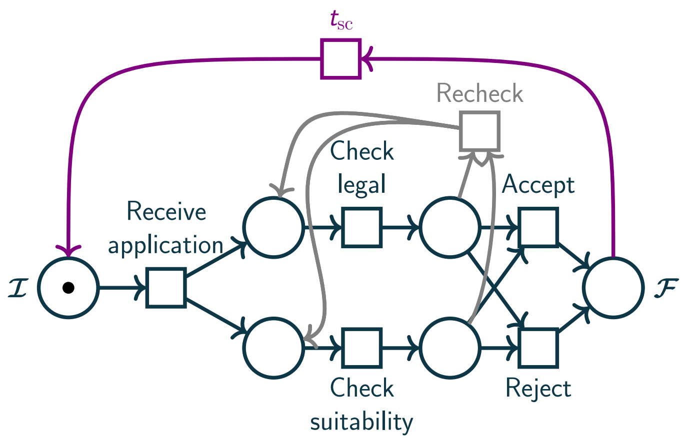
$\mathbf{m}' = \{\mathcal{F} \colon 1\}$
otherwise N_{SC} is unbounded



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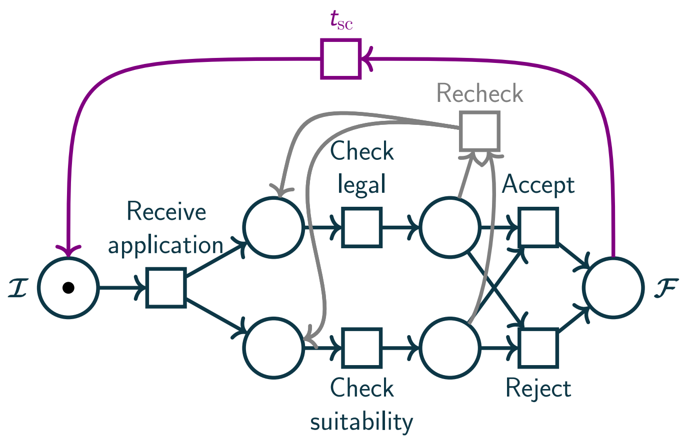

$\{\mathcal{I}\colon 1\}$ reaches $\mathbf{m}$ implies $\mathbf{m}$ reaches $\{\mathcal{I}\colon 1\}$

$\downarrow$

$\mathbf{m}$ reaches $\mathbf{m}'$ which marks $\mathcal{F}$

$\downarrow$

$\mathbf{m}' = \{\mathcal{F}\colon 1\}$ otherwise N_{SC} is unbounded

$\downarrow$

Any reachable marking can reach $\{\mathcal{F}\colon 1\}$

$(N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\})$ **is**

N **is 1-sound** $\Rightarrow$ **cyclic** $+$ **bounded**

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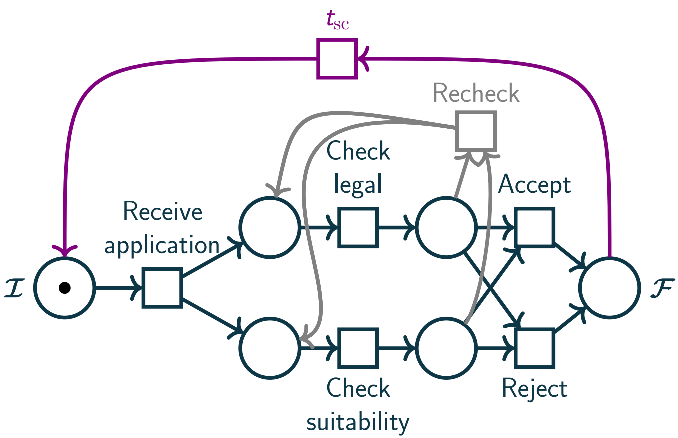

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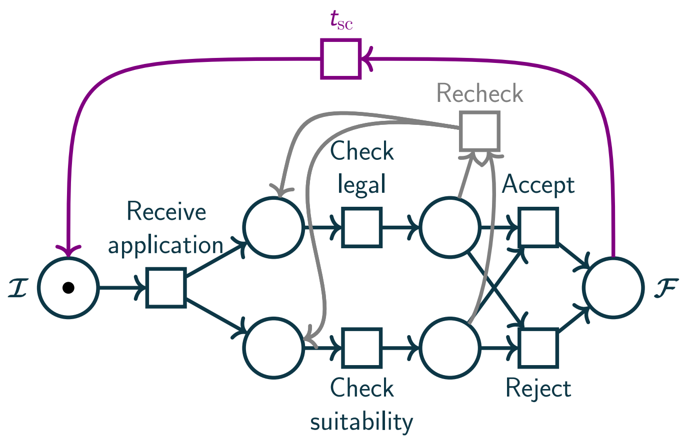

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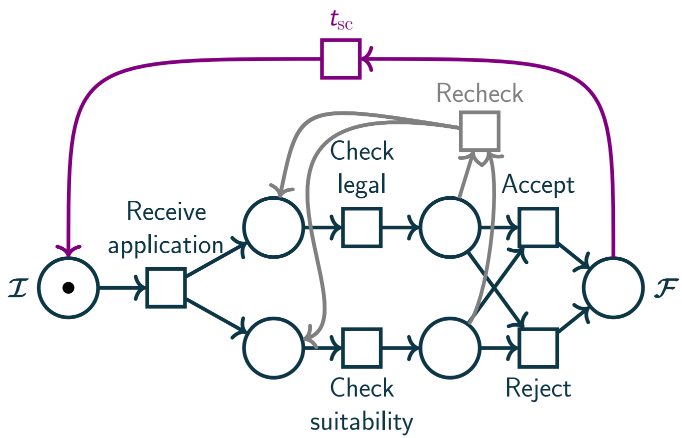

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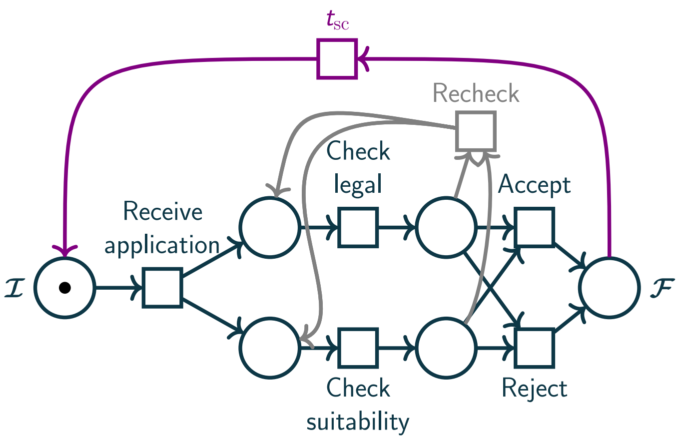

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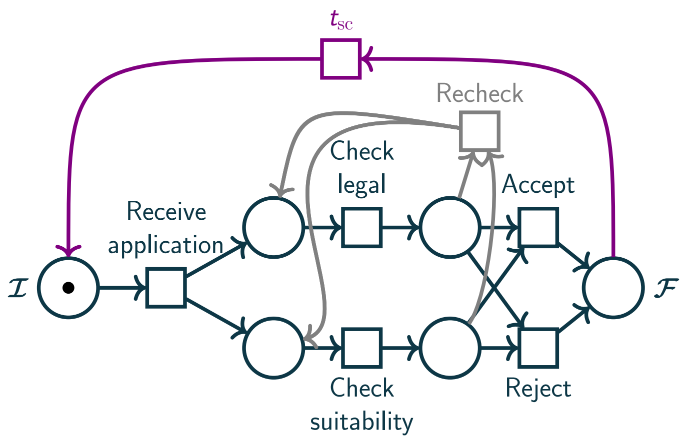

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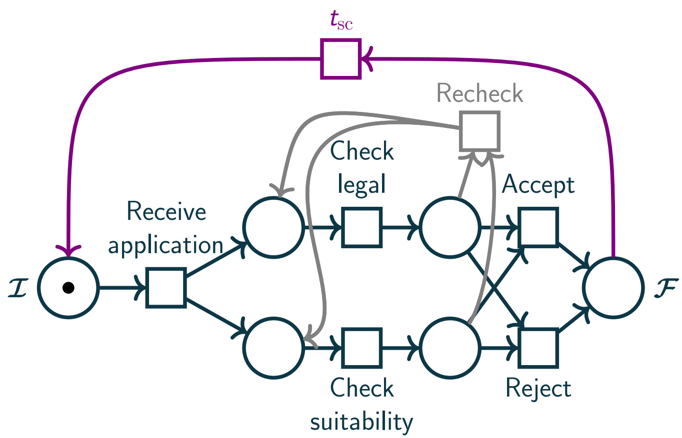
$\{\mathcal{I}\colon 1\}$ can reach $\mathbf{m}$ which can reach $\mathbf{m}' > \mathbf{m}$: $\mathbf{m}' = \mathbf{m} + \mathbf{n}$



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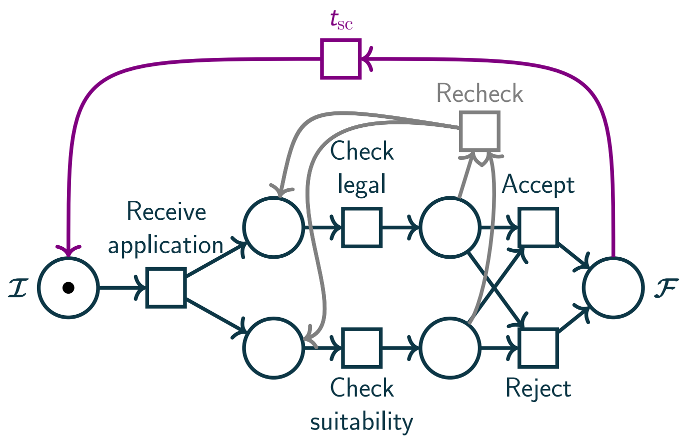

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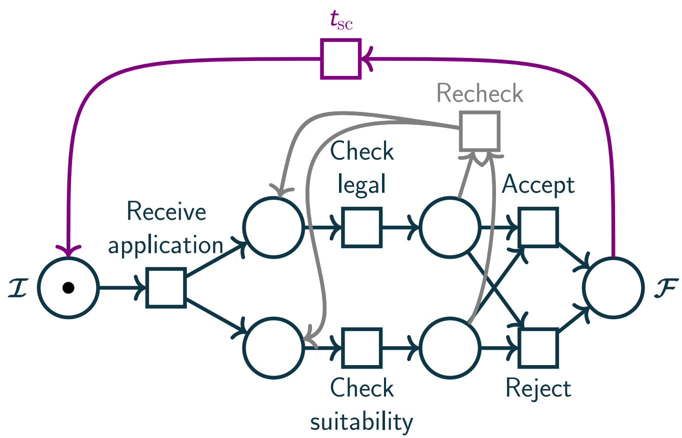

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$\{\mathcal{F}: 1\} + \mathbf{n}$ cannot reach $\{\mathcal{F}: 1\}$
$\Rightarrow N$ is not 1-sound!

N **is** 1**-sound** $\Leftrightarrow$ $(N_{\mathrm{SC}}, \{\mathcal{I}\colon 1\})$ **is cyclic** $+$ **bounded**

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In EXPSPACE
[Bouziane & Finkel, '97]

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Checking soundness - complexity?

	known results	our work	
***k*-soundness**	Decidable EXPSPACE-hard? [van der Aalst;'96, '97]	EXPSPACE-complete	1.
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$\mathbb{Z}$-unboundedness implies not generalised sound

Z-boundedness: $\forall k \exists \vec{b}$: $\{\mathcal{I}: k\} \rightarrow_{\mathbb{Z}} m > 0$ implies $m \leq \vec{b}$

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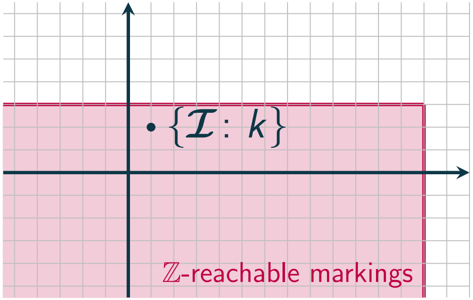

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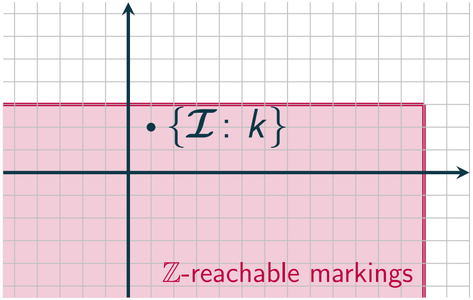

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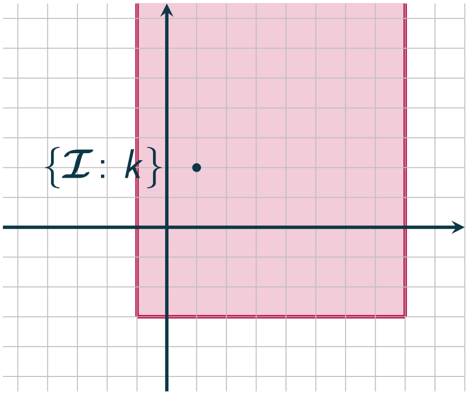

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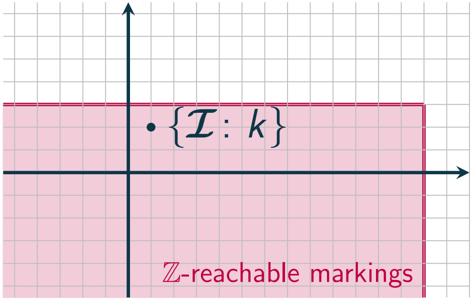

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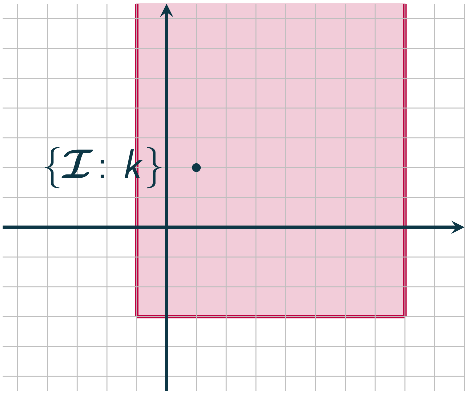

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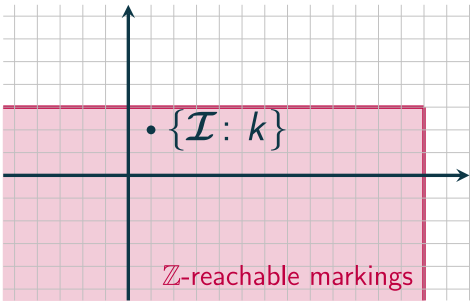

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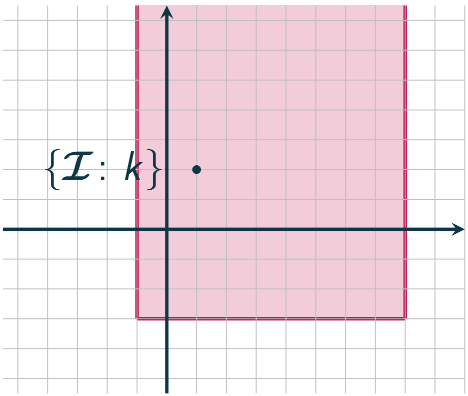

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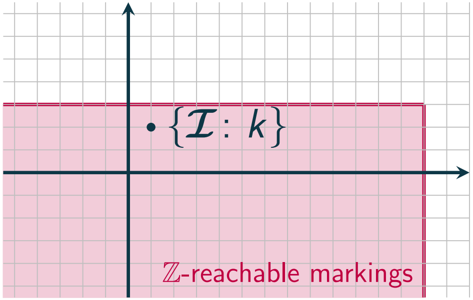

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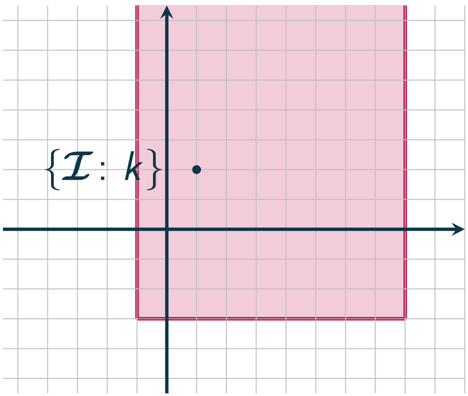

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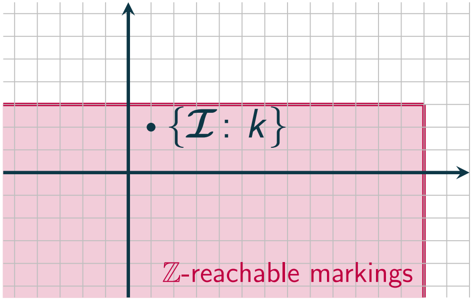

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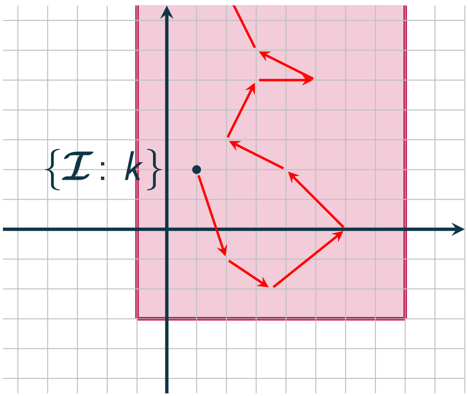

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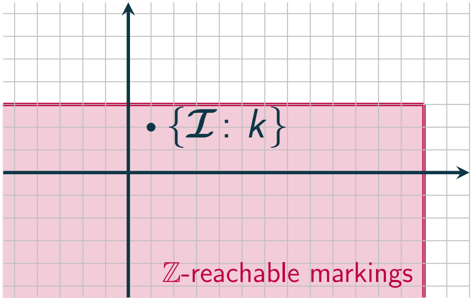

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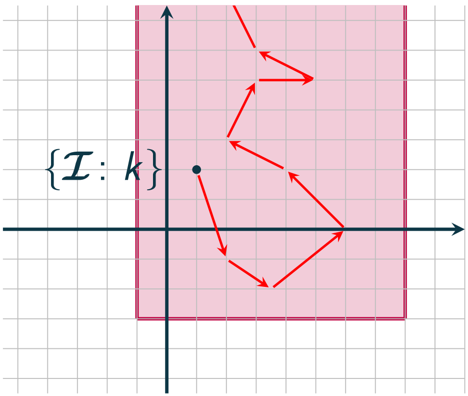

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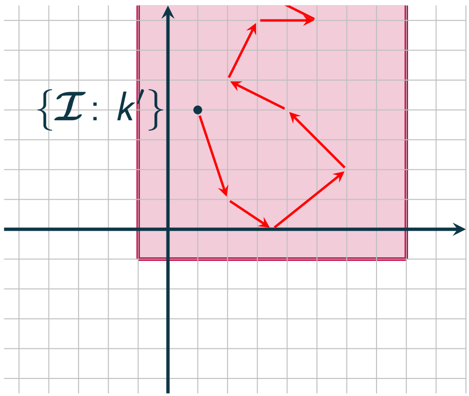

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$\{\mathcal{I} \colon k\} \xrightarrow{} \overset{\text{very large}}{m}$ Big markings must be reached by long runs

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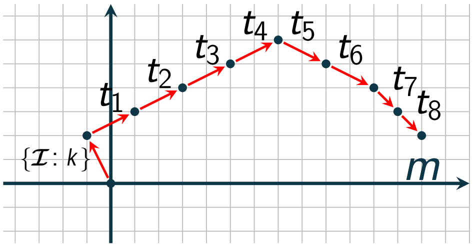

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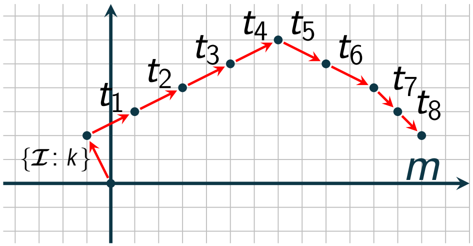

Steinitz Lemma:
Reorder vectors to stay close to straight line from $\vec{0}$ to m

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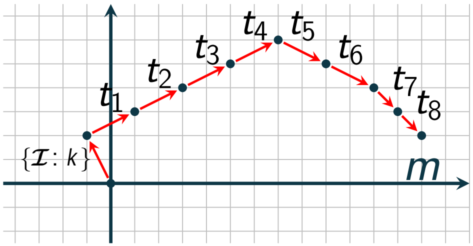

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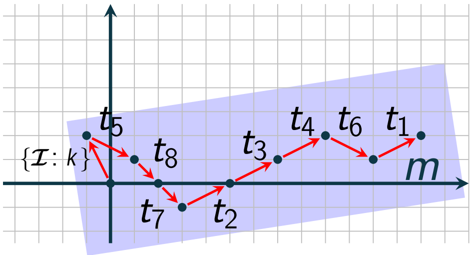

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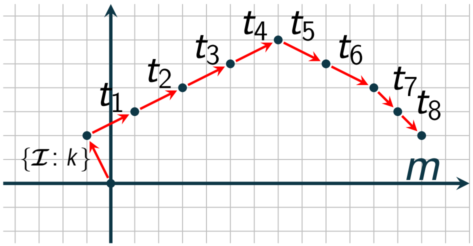

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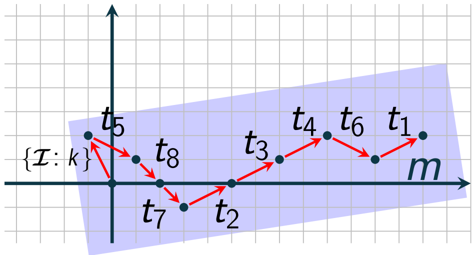

Long runs $\Rightarrow$ Many vectors $\Rightarrow$ Many points

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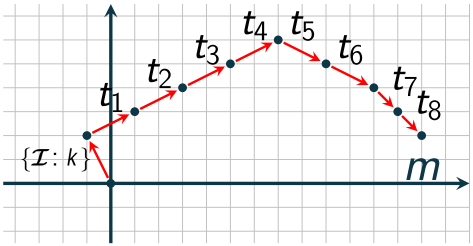

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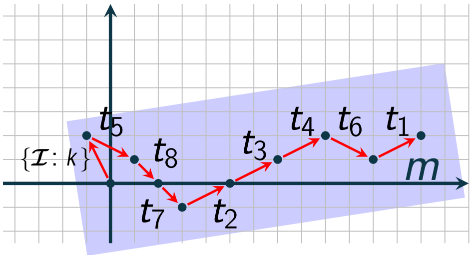

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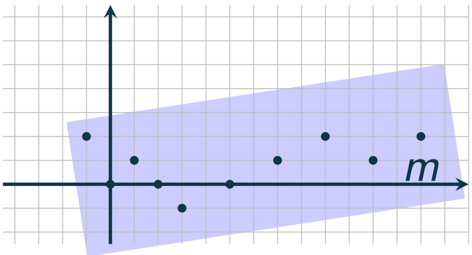
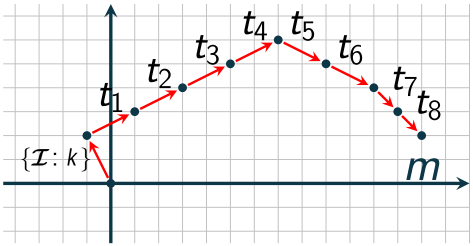


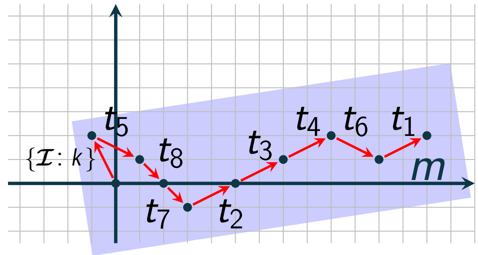
Big reachable markings imply $\mathbb{Z}$-unboundedness

$\{\mathcal{I}\colon k\} \xrightarrow{\text{very large}} m$

Big markings must be reached by long runs



Steinitz Lemma:
Reorder vectors to stay
close to straight line
from $\vec{0}$ to m



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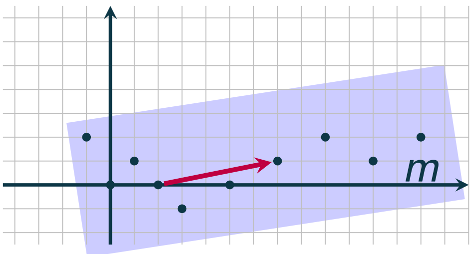

Enough points $\xRightarrow{\textit{Pigeonhole}}$
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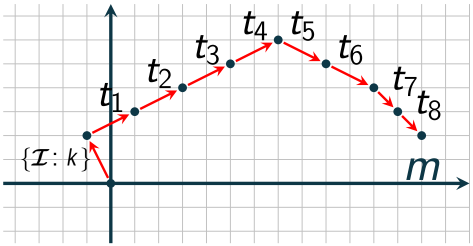

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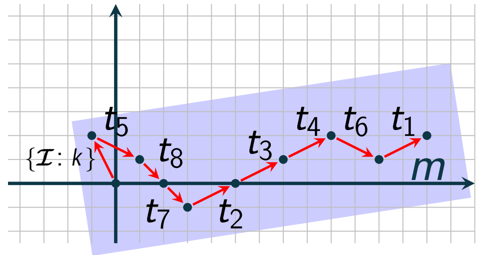

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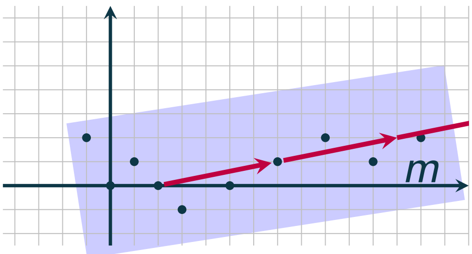

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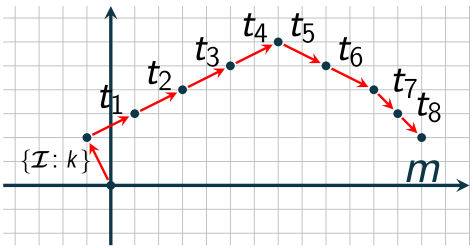

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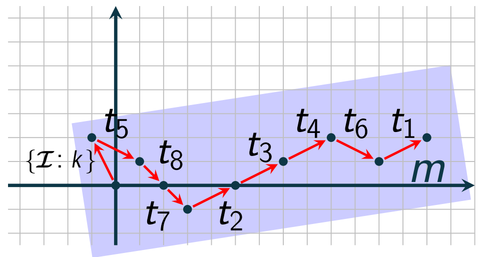

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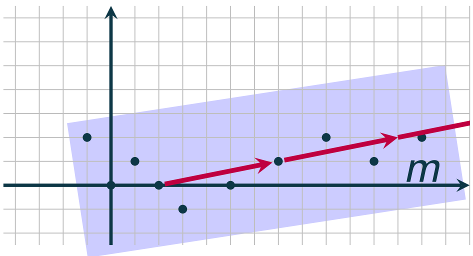

Enough points $\xRightarrow{\textit{Pigeonhole}}$ Strict increases $\Rightarrow$ $\mathbb{Z}$-unboundedness

Big reachable markings imply $\mathbb{Z}$-unboundedness!

Generalised soundness is in PSPACE

N is **generalised sound:**
$\forall k : \{\mathcal{I} : k\} \to m$ implies $m \to \{\mathcal{F} : k\}$

Witness k's are small: Not generalised sound $\Rightarrow$ unsound for a small k

A helpful necessary condition: Not $\mathbb{Z}$-bounded $\Rightarrow$ not generalised sound **1.**

Only enumerate small markings: Big marking reachable $\Rightarrow$ not $\mathbb{Z}$-bounded **2.**

Algorithm:

- Guess small k
- Check k-soundness: enumerate reachable markings
- If large markings are encountered: not generalised sound

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Checking soundness - complexity?

	known results	our work
***k*-soundness**	Decidable EXPSPACE-hard? [van der Aalst;'96, '97]	EXPSPACE-complete
Generalised soundness	Decidable [van Hee et al.;'04]	PSPACE-complete
Structural soundness	Decidable [Ţiplea, Marinescu;'04]	EXPSPACE-complete

Conclusion

Workflow nets formally model **processes**

Soundness is an intuitive correctness condition

Generalised soundness has connections to reachability over $\mathbb{Z}$